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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

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References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

ILIJAS FARAH. ***Analytic quotients.*** Memoirs of the American Mathematical Society vol. 148 no. 702, American Mathematical Society, Providence, R.I., 2000, xvi + 177 pp.

The Boolean algebra $\mathcal{P}\mathbb{N}$ of subsets of the natural numbers has some very well-known ideals, starting with the ideal $[\mathbb{N}]^{<\omega}$ of finite sets and the ideal $\mathcal{Z}$ of sets of asymptotic density zero; going a little farther we have the ideal $\mathcal{Z}_{\log}$ of sets $A \subseteq \mathbb{N}$ such that

$$\lim_{n\to\infty} \frac{1}{\ln n} \sum_{i\in A\cap n} \frac{1}{i+1} = 0.$$

For any such ideal we can consider the corresponding quotient Boolean algebra $\mathcal{P}\mathbb{N}/\mathcal{I}$. The algebra $\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}$ has long been recognised as one of the fundamental objects of set-theoretic analysis, topology and combinatorics. The others have not been studied so systematically, but show on the briefest of acquaintanceships the potential for generating fascinating questions. In this extraordinary monograph we are given some tools for tackling these questions which are surely going to be part of the essential kit for anyone working in the area. The most striking results concern the representation of Boolean homomorphisms between quotient algebras in terms of functions from $\mathcal{P}\mathbb{N}$ to itself or between cofinite subsets of $\mathbb{N}$.

The first steps are already far from being obvious; we need definitions which will lead to a useful classification scheme. One is well known. A *P-ideal* is an ideal $\mathcal{I}$ such that for every sequence $\langle I_n\rangle_{n\in\mathbb{N}}$ in $\mathcal{I}$ there is an $I \in \mathcal{I}$ such that $I_n \setminus I$ is finite for every n. The next is natural enough to the lateral thinker: an ideal $\mathcal{I}$ is *analytic* if it is an analytic

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subset of $\mathcal{P}\mathbb{N}$ when $\mathcal{P}\mathbb{N}$ is given its usual compact metric topology, that is, is identified with the Cantor set in $[0,1]$. Returning to the combinatorial aspect, $\mathcal{I}$ is *ccc over* $[\mathbb{N}]^{<\omega}$ if for any uncountable family $A \subseteq \mathcal{P}\mathbb{N} \setminus \mathcal{I}$ there are distinct a, $b \in A$ such that $a \cap b$ is infinite. Concerning homomorphisms between quotient spaces, let us say that a function $F\colon \mathcal{P}\mathbb{N} \to \mathcal{P}\mathbb{N}$ *represents* $\pi\colon \mathcal{P}\mathbb{N}/\mathcal{I} \to \mathcal{P}\mathbb{N}/\mathcal{J}$ if $\pi(a^\bullet) = (F(a))^\bullet$ for every $a \subseteq \mathbb{N}$. We shall be looking for representations F which are continuous, or have the Baire property, or are *asymptotically additive*, that is, for which there are disjoint sequences $\langle K_i\rangle_{i\in\mathbb{N}}$, $\langle L_i\rangle_{i\in\mathbb{N}}$ in $[\mathbb{N}]^{<\omega}$ such that $F(a) = \bigcup_{i\in\mathbb{N}} F(a \cap K_i) \cap L_i$ for every $a \subseteq \mathbb{N}$. A striking fact—easy to prove when you know how—is that a Boolean homomorphism between quotients has a continuous representation iff it has a representation with the Baire property.

Our starting point is the following remarkable result due to S. Solecki (***Annals of Pure and Applied Logic***, vol. 99 (1999) pp. 51–72): a proper ideal of $\mathcal{P}\mathbb{N}$ containing all finite sets is an analytic P-ideal iff it is of the form $\operatorname{Exh}(\nu)$ for some lower semi-continuous submeasure ν on $\mathcal{P}\mathbb{N}$. I see I have to give some more definitions. A *submeasure* on $\mathcal{P}\mathbb{N}$ is a functional $\nu\colon \mathcal{P}\mathbb{N} \to [0,\infty]$ such that $\nu(\emptyset) = 0$ and $\nu(a) \le \nu(a \cup b) \le \nu(a) + \nu(b)$ for all a, $b \subseteq \mathbb{N}$. It is *lower semi-continuous* if it is lower semi-continuous for the topology of $\mathcal{P}\mathbb{N}$, that is, if $\nu(a) = \lim_{n\to\infty} \nu(a\cap n)$ for every $a \subseteq \mathbb{N}$. Now $\operatorname{Exh}(\nu)$ is the ideal $\{a\colon \lim_{n\to\infty} \nu(a\setminus n) = 0\}$. If $\nu(a) = 1$ for every non-empty set a, $\operatorname{Exh}(\nu) = [\mathbb{N}]^{<\omega}$; if $\nu(a) = \sup_{n\ge 1} \frac{1}{n}\#(a \cap n)$, $\operatorname{Exh}(\nu) = \mathcal{Z}$.

One more definition. A submeasure ν on $\mathcal{P}\mathbb{N}$ is *entirely non-pathological* if whenever $a \subseteq \mathbb{N}$ and $\epsilon > 0$ there is an additive functional $\mu\colon \mathcal{P}\mathbb{N} \to [0,\infty[$ such that $\mu \le \nu$ and $\nu a \le \epsilon + \mu a$; it is easy to check that naturally arising submeasures generally have this property. Now I can state one of the new theorems from this memoir. If ν is an entirely non-pathological lower semi-continuous submeasure on $\mathcal{P}\mathbb{N}$, $\mathcal{I}$ is an ideal including $[\mathbb{N}]^{<\omega}$, and $\pi\colon \mathcal{P}\mathbb{N}/\mathcal{I} \to \mathcal{P}\mathbb{N}/\operatorname{Exh}(\nu)$ is a Boolean homomorphism, then π has a continuous representation iff there is an $h\colon \mathbb{N} \to \mathbb{N}$ such that $\pi a^\bullet = h^{-1}[a]^\bullet$ for every $a \subseteq \mathbb{N}$. The proof uses some remarkable calculations on finite sets.

The requirement that a homomorphism should have a continuous representation is visibly a strong one, and even after discovering that our favourite ideals are derivable from good submeasures most of us would expect the last theorem to have limited applicability. And indeed there are simple examples of homomorphisms, either injective or surjective, which do not have continuous representations. But we come now to the heart of the monograph. S. Shelah and J. Steprāns (***Proceedings of the American Mathematical Society***, vol. 104 (1988) pp. 1220–1225) showed that if the Proper Forcing Axiom is true then every automorphism π of $\mathcal{P}\mathbb{N}/[\mathbb{N}]^{<\omega}$ is trivial, that is, there are cofinite sets I, $J \subseteq \mathbb{N}$ and a bijection $h\colon J \to I$ such that $\pi a^\bullet = h^{-1}[a]^\bullet$ for every $a \subseteq \mathbb{N}$. Here we find that if PFA is true, $\mathcal{I}$ and $\mathcal{J}$ are analytic P-ideals, and $\pi\colon \mathcal{P}\mathbb{N}/\mathcal{I} \to \mathcal{P}\mathbb{N}/\mathcal{J}$ is a Boolean isomorphism, then π has a continuous representation; if $\mathcal{J}$ is of the form $\operatorname{Exh}(\nu)$ for some entirely non-pathological submeasure ν, then π is trivial. For these results we have to work hard. The key seems to be to look at the ideal $\mathcal{K}$ of those $a \subseteq \mathbb{N}$ such that the restriction of π to the principal ideal generated by a can be represented by a continuous function, and show that $\mathcal{K}$ is ccc over $[\mathbb{N}]^{<\omega}$, that a single continuous function can represent π on the ideal $\{a^\bullet\colon a \in \mathcal{K}\}$, that this function can be taken to be asymptotically additive, and that it then represents π.

It follows at once (if PFA is true) that among the naturally arising submeasures, different submeasures lead to different quotient algebras. (It was already known that under the continuum hypothesis the quotients are often isomorphic; see W. Just & A. Krawczyk, ***Transactions of the American Mathematical Society***, vol. 285 (1984), pp. 411–429.)

What I have tried to explain so far accounts for about half of the monograph. There is plenty more. In Chapter 2 the author explains why a precise pair of consequences of PFA (the Open Colouring Axiom and Martin's Axiom) should be the basis of the main result above. In Chapter 4 he discusses autohomeomorphisms of powers of the Čech-Stone compactification $\beta\mathbb{N}$ and the remainder $\beta\mathbb{N}\setminus\mathbb{N}$, showing that OCA + MA imply that all such autohomeomorphisms can be pieced together from elementary operations—an extension of the main theorem in the case $\mathcal{I} = \mathcal{J} = [\mathbb{N}]^{<\omega}$. In Chapter 5 he looks at gaps in quotient algebras and their preservation by homomorphisms.

I see that I have used rather a lot of formulae, corresponding to the fact that I have attempted to describe some very technical points. In my view it is in these technical points that the value of the monograph can be found. It is not easy reading, but it is indispensable, and there is much more to be got from these ideas—as, indeed, the author has already shown; see his papers *Rigidity conjectures*, pp. 252–271 in ***Proceedings of Logic Colloquium 2000*** (R. Cori, A. Razborov, S. Todorčević & C. Wood, editors), Association for Symbolic Logic, 2005 (Lecture Notes in Logic, vol. 19) and *Luzin gaps*, ***Transactions of the American Mathematical Society***, vol. 356 (2004) pp. 2197–2239.

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Krzysztof R. Apt. ***Principles of constraint programming.*** Cambridge University Press, Cambridge, United Kingdom, 2003, xii + 407 pp.

Many computational problems—from areas as diverse as combinatorics, computational algebra, scheduling, and temporal reasoning—naturally admit formulation as a constraint satisfaction problem (CSP). From a high-level viewpoint, a CSP involves deciding if there exists an assignment to a given set of variables satisfying a collection of requirements, called constraints. One may formally define a CSP as a finite set of variables V where each variable $v \in V$ has a set D_v called its domain associated with it, along with a set of constraints. Each constraint is a pair consisting of a variable tuple $(v_1, \ldots, v_k) \in V^k$ and a subset C of $D_{v_1} \times \cdots \times D_{v_k}$, and such a constraint is satisfied by an assignment f mapping each variable v to an element of D_v, if $(f(v_1), \ldots, f(v_k)) \in C$. The goal is to decide if there is a *solution*, an assignment to the variables satisfying all of the constraints.

Contents of book. This book is centered on methods for solving CSPs. The first three chapters constitute an introduction to the subject matter of the book. Chapter 1 gives a short survey of the area and of the book itself. Chapter 2 defines the constraint satisfaction problem and gives no fewer than fourteen examples of CSPs and classes of CSPs, which are convincing in terms of conveying the breadth of problems that can be placed into the CSP framework. In chapter 3, notions of equivalence between CSPs are defined and a generic skeleton procedure for solving CSPs, which is fleshed out in later chapters, is presented and discussed, along with examples. One main operation of this procedure is to perform *constraint propagation*, which here is an umbrella term for (typically efficient) methods that transform a CSP into another CSP that is simpler, but equivalent. Another main operation this procedure performs is "splitting", in which the CSP at hand is split into two or more CSPs having the property that the union of their solutions is (in some sense) equal to the solutions of the original CSP; the derived CSPs are then dealt with independently. This operation essentially gives the procedure the ability to perform a search over the space of possible assignments.

Chapter 4 first introduces a proof-theoretical framework which is used, in both this chapter and following ones, to describe transformations of CSPs by way of proof rules. The chapter then gives, for three different types of problems, *complete constraint solvers*, which the author defines to be a method transforming a given CSP into another CSP from which it is straightforward to either generate all solutions to the original CSP, or determine that none exist.

The next three chapters are concerned with notions of, and algorithms for achieving, *local consistency*. What is local consistency? There are many different notions of local consistency—with names like node consistency, arc consistency, and path consistency—and these can all be seen as properties of CSPs: that is, a CSP either is or is not node consistent depending on whether certain conditions hold. Generally, a notion of local consistency holds on a CSP if, from a certain localized view of the CSP, no inference can be performed. To give an example, a CSP is *hyper-arc consistent* if for every constraint C on the variables $(v_1, \ldots, v_k)$ and every index $i \in \{1, \ldots, k\}$, the projection of C onto the ith coordinate, that is, $\pi_i C = \{d_i \mid (d_1, \ldots, d_k) \in C\}$, is equal to D_{v_i}. If this is not the case, no assignment satisfying C can map v_i to a value in $D_{v_i} \setminus \pi_i C$, and the values in $D_{v_i} \setminus \pi_i C$ can be, in a certain way, removed from the CSP. Hyper-arc consistency can be established by iterating this removal step until the CSP is hyper-arc consistent. An intuitive way one might think of hyper-arc consistency is that it holds when the CSP is "consistent" from the local view of any individual variable. The algorithms given by the book for establishing different notions of local consistency serve as our main examples of constraint propagation methods for the mentioned skeleton procedure.

Chapter 5 defines a number of different notions of local consistency. For each notion of local consistency, proof rules are given and shown to characterize the consistency notion in the sense that a CSP obeys the consistency notion if and only if it is closed under all possible applications of the proof rules. Chapter 6 investigates the issue of how local consistency can be established on constraints that are presented in a particular syntactic form. Syntactic transformation rules are given for different classes of constraints, and in some cases proved to establish a form of local consistency. Chapter 7, which returns to the general perspective of chapter 5, concerns how to schedule the proof rule applications of a local consistency notion for efficiency, while still ensuring that closure under the proof rules is reached.

Chapter 8 deals with search algorithms for CSPs, with particular attention given to combining search with constraint propagation algorithms. Finally, chapter 9 discusses a variety of issues related to the previous chapters, such as modeling, implementation issues, alternative forms of search, and over-constrained problems.

Opinion. A unified book treatment of basic concepts and ideas of constraint satisfaction would be extremely welcome, particularly to help those not familiar with the area approach the contemporary literature. The book at hand aims at such a treatment. Unfortunately, I encountered extreme problems with the writing and presentation of this book, that I believe will substantially hinder its usage. In the remainder of this review, I will first illustrate these problems by focusing on a single concrete example, and point out other instances of these problems. I will then discuss problems of a more technical nature, address two specific points of imprecison, and close.

Let me begin by considering the concrete example of the opening sentences of chapter 5, the first chapter on local consistency:

> Ideally, we would like to solve CSPs directly, by means of some efficient algorithm. But the definition of a CSP is extremely general, so, as already mentioned in Chapter 1, no universal efficient methods for solving them exist. Various general techniques were developed to solve CSPs and in the absence of efficient algorithms a combination of these techniques is a natural way to proceed.

If we are proceeding in the absence of efficient algorithms, one expects that any algorithms encountered during our procession ought to be evaluated from the standpoint of efficiency. I found essentially no discussion of efficiency issues in chapters 5, 6, and 7 (the chapters on local consistency) and no form of runtime analysis of the algorithms in chapter 7. References to notions of efficiency are also placed prominently in the initial sentences of other chapters: the second sentence of chapter 4 states that "in practice we are interested in efficient constraint solvers", and the first sentence of chapter 6 refers to "efficient solving methods". The ultimate result of the emphasis on efficiency that one perceives from these references and their placement, along with the lack of actual discussion of efficiency issues, is an undermining of the reader's confidence that she understands even what the book is about or trying to achieve.

These references to efficiency not only induce confused expectations about what we are about to read, but cannot be understood on their own terms. Despite the repeated use of the adjective "efficient", the author never actually defines what he means by this word. I certainly appreciate that the notion one is after here might not be formally definable, but not even a hint is dropped as to what the author's conception is. This is all the more worrisome because there is, in computer science, a well-established mathematical notion of efficiency—that of polynomial-time computation—with which it appears that the author's conception of efficiency does not coincide (at least, from the cited sentence of chapter 4, along with the fact that there are polynomial-time algorithms that are not usable in practice), and with which the author would have done well to compare his notion.

Moreover, the author fails to define what he means by *universal* efficient method, and I could not find any discussion of their nonexistence in chapter 1.

The preceding discussion of three sentences from this book begins to convey some of the problems I experienced throughout my reading of it. Topics are not motivated nor framed well: for instance, chapter 4 defines the notion of a complete constraint solver and then presents such solvers for some particular problem domains, but does not even allude to the obvious question of whether or not there are problem domains without complete solvers. It is difficult to anticipate what is coming next: for example, the first section of chapter 7 gives a completely abstract development of various iteration algorithms, which are all fully developed before even the simplest is instantiated and applied to constraint satisfaction. Terminology is sometimes not defined well or used unclearly: compare the descriptions of "constraint propagation algorithms" on pages 70 and 254, not to be confused with "constraint propagation" on pages 66–67. Ideas are alluded to but not followed up on: chapter 7 does not transparently answer the three motivating questions in its introduction, and chapter 1 makes reference to "connections between rule-based programming and constraint programming", which are never made clear. The writing and presentation at times seem assumption-laden: why is a combination of techniques natural, in the above quote? And what justifies the skeleton procedure of chapter 3?

At a more technical level, I also found the mathematical definitions, notation, and conventions of this book problematic. In particular, they are expressed unclearly, highly counterintuitive, and employed inconsistently. As an example, let us examine the author's definition of constraint satisfaction problem:

> Consider a finite sequence of variables $Y := y_1, \ldots, y_k$ where $k > 0$, with respective domains $D_1, \ldots, D_k$ associated with them. . . . By a *constraint* C on Y we mean a subset of $D_1 \times \cdots \times D_k$. . . . By a *constraint satisfaction problem*, or a CSP, we mean a finite sequence of variables $X := x_1, \ldots, x_n$ with respective domains $D_1, \ldots, D_n$, together with a finite set $\mathcal{C}$ of constraints, each on a subsequence of X.

First, note that the author insists that the variables of a CSP fall into a sequence, but this does not seem crucially used until possibly the second-to-last chapter. Next, observe that each constraint in a CSP is on a subsequence of the variable sequence, but it does not appear that the chosen subsequences are part of the CSP; in the case that the variables of a CSP have common or overlapping domains, it can then be impossible to know, given a constraint of a CSP, which subsequence of variables it is on. The "variables of a constraint" are generally specified somehow, but not in any consistent way; in some of chapter 5, they are not specified with the constraint, and as a result, even the simplest proof rule (node consistency) is not self-contained. Another issue with these definitions is that the domains of variables are sometimes shrunk without changing the constraints (for example, in Example 4.6, or at the beginning of chapter 5.5). In general, doing this may result in an object that is technically not a CSP, according to the above definition—because a constraint C satisfies a containment of the form $C \subseteq D_1 \times \cdots \times D_k$, which may fail to hold if one of the domains D_i is shrunk. To be sure, after the definition of CSP, a note is made about how constraints and domains are specified in a syntactic language, and how "it is implicit that each constraint is a subset of the Cartesian product of the associated variable domains". The point seems to be that a "constraint" C on variables $y_1, \ldots, y_k$ with domains $D_1, \ldots, D_k$ ought to be interpreted as $C \cap D_1 \times \cdots \times D_k$. In this case, the containment $C \subseteq D_1 \times \cdots \times D_k$ in the definition of a constraint is really just a consequence of an implicit assumption, and not appropriate as part of the actual definition. Should not more thought have been given to these central definitions?

I also identified two specific problems of a more subtle character. First, in chapter 7.1.3, three generic algorithms are presented, and for each sufficient conditions are identified under which they work. On page 272 and also on page 279, it is suggested that, for a notion of local consistency, two of the generic algorithms cannot be employed, because the sufficient conditions are not met. There is a logical flaw here, namely that not satisfying the sufficient conditions does *not* logically imply that the algorithms do not work. Second, on pages 254–255, examples are given of cases where a CSP can be "solved solely by means of a constraint propagation algorithm". One example is a CSP representing a crossword puzzle, to which an arc consistency algorithm is applied; in the resulting CSP, "each variable domain is reduced to a singleton set and consequently this CSP is solved" (the author's words, page 143). The author does not define what he means by "solved" in these contexts, but it appears that he is getting at the fact that it is straightforward to generate a solution to the resulting CSP (see the discussion at the end of chapter 5.2). Another example is the class of CSPs having bounded "width" (from Theorem 5.48, page 173), but this example seems different in nature. While establishing k-consistency certainly gives a procedure for deciding whether or not such a CSP has a solution, there is not any discussion of how one might actually construct a solution to a CSP from this class (at least, up to the point where the examples are given). The potentially relevant question of how one might determine if a CSP has bounded width is also undiscussed.

As this sampling of my problems with the book should suggest, I found the book extremely difficult to follow on many different levels. At a broad level, one cannot discern the relative importance of the topics covered, nor an overall conception and justification of the subject matter. Those already acquainted with the subject who are both willing to tolerate and able to compensate for the book's defects may be able to use it as a reference; for others, I fear that the book's habitual lack of clarity, precision, and organization render it unusable.

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The law of non-contradiction: New philosophical essays, edited by Graham Priest, J. C. Beall, and Bradley Armour-Garb, Oxford University Press, Oxford, 2004, xii + 443 pp.—therein:

J. C. Beall. *Introduction: At the intersection of truth and falsity*. Pp. 1–19.

Graham Priest. *What's so bad about contradictions?* Pp. 23–38.

Ross Brady. *On the formalization of the law of non-contradiction*. Pp. 41–48.

Patrick Grim. *What is a contradiction?* Pp. 49–72.

Greg Restall. *Laws of non-contradiction, laws of the excluded middle, and logics*. Pp. 73–84.

R. Mark Sainsbury. *Option negation and dialethesias*. Pp. 85–92.

Achille Varzi. *Conjunction and contradiction*. Pp. 93–110.
Bradley Armour-Garb. *Diagnosing dialetheism*. Pp. 113–125.
Bryson Brown. *Knowledge and non-contradiction*. Pp. 126–155.
Otávio Bueno and Mark Colyvan. *Logical non-apriorism and the 'law' of non-contradiction*. Pp. 156–175.
David Lewis. *Letters to Beall and Priest*. Pp. 176–177.
Michael Resnik. *Revising logic*. Pp. 178–194.
J. C. Beall. *True and false—as if*. Pp. 197–216.
Jon Cogburn. *The philosophical basis of what? The anti-realist route to dialetheism*. Pp. 217–234.
Jay Garfield. *'To pee or not to pee?' Could* that *be the question? (Further reflections of the dog)*. Pp. 235–244.
Frederick Kroon. *Realism and dialetheism*. Pp. 245–263.
Edwin Mares. *Semantic dialetheism*. Pp. 264–275.
Vann McGee. *Ramsey's dialetheism*. Pp. 276–291.
Laurence Goldstein. *The barber, Russell's Paradox, Catch-22, God and more: A defence of a Wittgensteinian conception of contradiction*. Pp. 295–313.
Greg Littmann and Keith Simmons. *A critique of dialetheism*. Pp. 314–335.
Stewart Shapiro. *Simple truth, contradiction, and consistency*. Pp. 336–354.
Neil Tennant. *An anti-realist critique of dialetheism*. Pp. 355–384.
Alan Weir. *There are no true contradictions*. Pp. 385–417.
Edward Zalta. *In defense of the law of non-contradiction*. Pp. 418–436.

The 'law of non-contradiction' (LNC) is perhaps $\neg(A \wedge \neg A)$, or maybe '$(A \wedge \neg A)$ cannot be true', or maybe 'no statement and its negation can simultaneously be true', or maybe 'A cannot be simultaneously true and not true', or maybe 'A cannot be simultaneously true and false', or maybe 'one cannot both affirm and deny A'. (And there are other possibilities surveyed by Grim in his article. He says there are approximately 240 different versions on offer.) This collection contains 23 essays plus introduction, all but one newly published, by most of the major players in the debate over whether LNC is correct. (A debate that many readers will be surprised to hear exists at all!) If LNC is not correct, then there are some instances of it that are incorrect—in whatever version of the law one chooses to work with. Priest, in his entry in this collection (a reprint of his 1998 ***Journal of Philosophy*** article), emphasizes the 'some' in this formulation: disbelievers in LNC are *not* required to think that *all* instances of LNC are false, and indeed, various of the articles (including Priest's) *are at pains to explain* that they think that "almost all" of these instances are in fact true (and not also false). The 'in fact' here is an important ingredient of the general position advocated by Priest and others in this collection (and explicitly argued in Bueno and Colyvan, in Garfield, and in Brown): there is no *a priori* warrant or justification for this or any other logical principle, but rather they are to be held up to empirical justification. In the case of LNC, according to Priest and other detractors of LNC, the empirical evidence shows that there are false instances. Throughout this collection, the most clear counterexamples are thought to be the paradoxes: the liar sentence (which they claim to be both true and false) , the paradox of the preface (where you believe of each factual statement you wrote that it is true but also believe on the basis of past experience that at least one is false), and so on. (Armour-Garb's contribution concerns how they can be still called paradoxes by dialetheists, since they believe that the argumentation is valid which leads to the conclusion that they are both true and false). Priest also believes that certain other statements should be seen as both true and false, such as the case of walking through a doorway. In such an event, there will be a time when one's center of mass is precisely at the center of the doorway. At that moment, says Priest, the sentence 'I am in the room and not in the room' will be true. (One might note that I have shifted among various versions of LNC, sometimes using 'true'/'false', sometimes using negations with a conjunction, and sometimes using a large set of separate sentences whose consequences include a contradiction in one of the other senses.) It is, of course, a bit strange to talk of LNC being false in the logics proposed by Priest and others, since $\neg(\text{A} \wedge \neg\text{A})$ is a theorem of these logics. But this shows that some theorems can be false—so long as they are also true!

Those who believe that the LNC has counterexamples—that is, those who believe that there are some true contradictions—are called *dialetheists*. These are to be distinguished from *paraconsistentists*, who hold only that logic should not be "explosive" (there should be no formula φ such that φ implies every sentence). Of course, classical logic (a term I will continue to use despite Priest's allegations that it is a very recent invention) is explosive because a contradiction implies every sentence, and so naturally dialetheists are paraconsistentists in their logic. But the converse is not true, and in fact there are many "relevantists" (those who believe that some form of relevant logic—all of which are non-explosive—is The One True Logic) who are not dialetheists. Some relations between paraconsistentists and dialetheists are discussed in Mares' contribution, and various ins and outs of the relationships among some relevant logics and dialetheism are discussed in Restall's contribution. Restall's article also develops an idea brought up in Priest's article concerning the distinction between the assertion of $\neg\text{A}$ and the denial of A. This is a theme that runs through many of the articles in the collection. The question concerns whether, even if one way of putting LNC might have exceptions, does another way of putting it nonetheless obey LNC? Perhaps dialetheists are committed to having one form (the assertion/denial form maybe) of LNC true (and not also false), such as "One cannot assert A and deny A simultaneously"? If there is such a form that is true (and not also false), then might these forms themselves be susceptible to a paradox that the dialetheist cannot admit, even though they can allow some contradictions? For example, "I hereby deny this very assertion." This general topic is explored in the article by Littmann and Simmons, who offer many classes of possible "inconsistencies" in dialetheistic logic—inconsistencies that they claim dialetheism cannot accept despite their general acceptance of (some) inconsistencies.

David Lewis, like Aristotle, thought LNC to be the most fundamental logical principle. In letters to Priest, parts of which are included in this volume, he says that a dispute "needs common ground" but that "in this case, the principles *not* in dispute are so very much less certain than non-contradiction" that no real dialogue is possible. Lewis's view is attacked by Bueno and Colyvan, who argue for "logical pluralism" and for the view that even though Principle A might be less certain than Principle B, nevertheless Principle A may play a role in overthrowing Principle B, especially if there are many such less certain Principles. (One wonders what to make of the fact that there is a book with 101 "proofs" that $1 = 2$, and yet probably none of us has seen more than one proof that $1 \neq 2$. It does not seem that *any* number of invalid proofs could overturn one valid proof; yet this seems to be what Bueno and Colyvan are in essence suggesting.)

Aristotle thought that LNC was "the most secure principle" and could therefore not be proved. But he thought that a variety of ad hominem arguments could be mounted against one who claimed that it was false. "Let him just pick out some one definite substance. Let him for example say 'that is a man'. To be a man is to be some one object that has a certain set of properties that define what it is to be a man. But if there is some one object here with those properties, a person who says the opposite would of necessity be speaking falsely. He would be no better than a vegetable." (I paraphrase an argument at *Meta*. Γ 4 1006^a12ff. *Meta*. Γ 4 contains a series of such arguments which have been thought over the millennia to carry the day. Maybe Aristotle here could be accused of saddling the dialetheists with the view that *all* contradictions are true. Perhaps dialetheists will instead find those cases where a person "picks out some one definite thing" as always obeying LNC, thereby thwarting this ad hominem argument.)

Many of the articles in the book mention the perceived difficulty of arguing with a dialetheist. Under classicalist standards, one shows that an opponent in a dispute is wrong about X by showing that X conflicts with some other things that the opponent agrees with. But, so the difficulty alleges, the strongest way to demonstrate a conflict is to show X is inconsistent or contradicts what is independently believed. Yet the dialetheist will not be bothered by this, according to this difficulty, because a dialetheist might *embrace* the contradiction. Actually, there are many middle roads here, since after all a dialetheist need not accept *every* contradiction. (Priest thinks that contradictions have *prima facie* implausibility). But nonetheless, it is true that a dialetheist *could* embrace the contradiction, so just pointing to a contradiction in his belief set does not guarantee that he will feel required to change some of his beliefs. The argument form that is relevant to arguing with a dialetheist might instead be one of simplicity: since X conflicts with your beliefs, the simplest solution for you is to dispense with X rather than try to accommodate it.

Most modern analytic philosophers react to proposals that LNC might be false either as cases where some feature of evaluation has not been kept constant or as cases where the meaning of some logical word is being arbitrarily changed. Consider the first, cases like 'John is a diplomat but not a diplomat' (said of the very diplomatic John, who nonetheless has no government job), 'This blob is red and not red' (said of the halfway-to-pink blob), and 'Mary knows that it is her turn to drive in the carpool, but of course she doesn't (really) know that' (where "the standards of knowing" change from one conjunct to the next). Of this sort of case Aristotle said (*Meta*. Γ 3) " ... the same attribute at the same time to the same subject in the same respect, and we must also suppose whatever further qualifications are needed to guard against these trivial and eristic objections". The classicalist really need not feel any angst about this type of alleged contradiction, even though he could admit that if a dialetheist were to convince us on other grounds that some contradictions are true then that category of true contradiction might be extended so as to include these too.

The other reaction is to think that some logical word is being used wrongly. "If $(A \wedge \neg A)$ is true," the objection might run, "then either it can't imply $\bot$, or you have changed the meaning of $\bot$, or you are not using $\wedge$ to mean 'and', or you are not using $\neg$ to mean 'not'. Various of the articles in the book touch on the issue of what sort of negation dialetheists employ (Brady, Grim, Sainsbury, and Shapiro), which is clearly of importance in understanding their view. (Shapiro in particular claims that there is no negation that will do the job that the dialetheist wants, although Beall in his Introduction claims that this result only applies to the specific logic, LP, proposed by Priest, and not against other dialetheistic logics. Shapiro's article may be the one in the collection that is of most interest for its results of logic.) But not many discuss whether the dialetheist's $\wedge$ means *and* (although see McGee's discussion of Ramsey-style views on $\wedge$, where you cannot infer $A \wedge B$ given A and B as premises, and Varzi's discussion of collective vs. distributive *and*); and it is mainly Tennant here who questions the dialetheist about $\bot$. (Tennant claims that the dialetheist is committed either to saying that $\bot$ is sometimes true or that the dialetheist's logic will fail to transmit truth precisely in those cases where there is an allegedly true contradiction as a premise. Either of these is bad news for the dialetheist, says Tennant.) Weir's article discusses the general view that dialetheists are somehow changing the meaning of some one or other of the logical words; he argues that while classicists should take dialetheism seriously, in the end it is not right. Related to these papers, and to the dialetheist's attempt to establish "a new logic" that gives sense to contradictions being sometimes true, is Resnick's contribution on what it would mean to "revise logic" in a Quinean 'web of belief' framework. Goldstein tries to interpret contradictions in a Wittgensteinian way; Garfield brings forth the sort of "practical reasoning" that would lead one to adopt some sort of dialetheism; Brown offers a "preservationist" account of consequence which he claims is more general than dialetheistic accounts and which provides a rationale for adopting "logical pluralism"; and Zalta defends

LNC against dialetheistic arguments by means of his distinctive demarcation of two types of predication: exemplification vs. encoding. In his view the various paradoxes can be given as nice a treatment within his logic as they can in dialetheist logics (one version of which is set out in detail in Beall's contribution).

We are told by Cogburn that Dummettian anti-realists (who take verifiability seriously) not only can but must be dialetheists. On the other hand, Tennant assures us that all intuitionistic anti-realists (like him) must deny dialetheism. Kroon says that a realist who is a dialetheist must be willing to at least countenance a world in which *all* contradictions are true; and that to the extent that a one accepts dialetheism, one must not be a realist about the part of reality that generates true contradictions. Evidently the metaphysics behind dialetheism is not yet settled!

This collection is mandatory for anyone interested in adopting, rejecting, or even just understanding dialetheism. But subparts of it are also very useful collections of articles that would help one understand relevant logics, or negation, or the role of logic in a formal theory of language. The book is edited from the point of view of dialetheism, as evidenced in Beall's very substantial introduction, and is not at all a dispassionate discussion of the pros and cons of dialetheism (despite the section entitled "For the LNC"). But it is still the clearest and most accessible work that describes dialetheism in such a way that proponents and opponents can see what it is, and where its strong ridges stand and its weak underbellies lie.

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Heinrich Scholz. Logiker, Philosoph, Theologe. edited by Hans-Christoph Schmidt am Busch and Kai F. Wehmeier, Mentis, Paderborn, 2005, 134 pp.

The biography of Heinrich Scholz is remarkable with respect to his scientific development as well as concerning his personal integrity, most notably during the NS-time. Scholz had an extraordinary academic career: he started as a professor of philosophy of religion then went on to obtain a professorship in philosophy and then afterwards and until his retirement he was professor for logic and foundational studies in mathematics. He founded the famous "Institut für mathematische Logik und Grundlagenforschung" (ILG) at the university of Münster. Without exaggeration it can be stated that we have at least to owe it to Scholz that foundational studies in logic and mathematics after World War II were institutionally possible. Of course, Scholz himself was surely not a brilliant logician, but (especially for younger colleagues) he promoted an adequate academic atmosphere and the institutional frameworks necessary for it. It should be mentioned that the ILG is one of few institutes for foundational studies in mathematics and logic in Germany up till now which has worked on the highest international level. The named volume is dedicated to the person as well as to the scientist Heinrich Scholz. Therefore, the book contains not only contributions dealing with (philosophy of) mathematics or logic, but also papers on Scholz's philosophy of religion and metaphysics. The contributed papers were communicated at an international scientific conference on the occasion of the 50th anniversary of the ILG in Münster (23rd to 25th March 2000).

Despite his scientific development from theology via philosophy to logic, Scholz was still interested in the philosophy of religion and the history of theology. Therefore, in his paper "Ein standfester Mensch" (pp. 13–45), Arie Molendijk investigates the influences of the Christian faith and protestant theology on Scholz as a person and as a scientist. For most who know Scholz (only) as a philosopher and a logician, Reiner Wimmer focuses, from the reviewer's point of view, on a neglected and "exotic" field of his research: the philosophy of religion. In "Die Religionsphilosophie von Heinrich Scholz" (pp. 47–68) Wimmer gives a vivid overview of Scholz's contributions to this scientific field and shows what can be done within his philosophy of religion. For all who want to get a lucid account into this topic, Wimmer's contributions meet the needs. In "Heinrich Scholz als Metaphysiker" (pp. 69–83) Volker Peckhaus focuses on the development of Scholz's understanding of language and his method in philosophy—a development closely related to his investigations in metaphysics. Peckhaus argues against the claims of Eberhard Stock that Scholz's development in these aspects was rather discontinuous. Starting from this insofar "unknown Scholz" Peckhaus outlined some consequences for the later Scholz, especially for his preference of a rational symbolized language ("Leibnizsprache") and his metaphysical thinking. In "Scholz about *cogito, ergo sum*" (pp. 85–92) Jan Woleński investigates the inferential structure underlying the cogito argument by Descartes. Since Scholz had already interpreted the cogito argument, Woleński questioned the status of the "ergo". Using insights by Scholz as well as those by Łukasiewicz and Hintikka, Woleński concludes two results. First, the conclusion "I think" is derived as a necessary result from the premise. But since the premise is empirical, the conclusion is not purely necessary. For the reviewer the most interesting contribution concerning the historiography of the life and work of Scholz within the book under review is "Es ist die einzige Spur, die ich hinterlasse" (pp. 93–101) by the editors. Hans-Christoph Schmidt am Busch and Kai F. Wehmeier reconstruct in detail the way the ILG progressed from the very beginnings in 1928 until the final institutional setting in 1950. In all, it took five stages until the ILG could grow into a full blooded institute. In 1936 the formally known "Section B" of the Department ("Seminar") of Philosophy was upgraded to the logic section ("Logistische Abteilung") of the department. The next step consisted in the establishment of the Chair for Mathematical Logic and Foundational Studies in 1943. Followed by the establishment of the Department of Mathematical Logic and Foundational Studies ("Seminar für mathematische Logik und Grundlagenforschung") in 1946, the final step was taken in 1950. Additionally and remarkably, the used and, in parts, quoted correspondence between Scholz and the responsible ministerial offices over a period of more then twenty years give some very interesting insights into the relation between the scientist and the authority. In his article "Heinrich Scholz between Frege and Hilbert" (pp. 103–117), Göran Sundholm explains the position of Scholz with respect to Frege's epistemological view on logic and Hilbert's formal-axiomatic account starting from the point when Scholz entered the discussion in the early 1920s. Sundholm makes it intelligible that Scholz is, with respect to the view on content of formal languages and the status of the axioms (Aristotelian view vs. Hilbert's conception), to be placed "between" Frege and Hilbert. Influenced by Fichte and applying the Fichtian dichotomy between act and object within the foundations of mathematics, for Scholz we have to decide between Intuitionism and Platonism. Whereas Scholz also had a balanced view of Intuitionism, he discussed the topics strictly from a Platonist point of view. With Frege Scholz remains insistent on content in logic, whereas he prefers Hilbert's notion of a hypothetical axiom. For this Scholz used Tarski's set-theoretical semantics (as content) for adopting the notion of relative truth (satisfaction of axiom schemes). The author concludes that Scholz's position can be summarized by the catchword "he can be seen as the solution of the equation $\text{Frege}/X = X/\text{Hilbert}$." In their second paper "Heinrich Scholz und Jan Łukasiewicz" (pp. 119–131), the editors used some correspondences to show how Scholz supported and protected Łukasiewicz during the NS-time in Poland. Scholz's indisputably risky activities was aimed at successfully moving Łukasiewicz and his wife from Warsaw to Münster. With respect to the opening paper by Molendijk the last paper adds and completes a multidimensional view on the character and the person Heinrich Scholz.

The volume under review is full of insofar unknown aspects concerning Scholz and remarkable insights into his scientific career as well as his successes in establishing a rebirth of foundational studies in logic and mathematics in Germany. For all who know Scholz only as a philosopher of logic and mathematics, the contributions concerning his person, metaphysics and philosophy of religion vividly complete the picture of an outstanding scientific biography.

The papers—dealing with logic in a wider sense—are to be seen as helpful and appropriate contributions to the history of logic, especially for the history of logic in Germany after the great times of Frege and Hilbert. Whereas Scholz is to be seen in the "second row" behind the likes of Frege or Carnap, the volume under review compensates for the resulting neglect to a certain degree.

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VINCENT F. HENDRICKS. ***Logical lyrics: From philosophy to poetics.*** King's College Publications, London, 2005, xiii + 173 pp.

This book is a heterogeneous collection of citations and aphorisms on logic and logical matters. It is a follow-up to ***Feisty Fragments for Philosophy*** published a year before by Vincent F. Hendricks with the same publishing house. Logic is often understood in various ways, going from simple common sense to a technical science accessible only to specialists. This book browses all these interpretations by collecting citations of all sorts of people, from Joseph Addison to Emile Zola via Isaac Asimov, Gottlob Frege and Alfred Hitchcock.

The almost five hundred citations are often informative and humorous. Many of them had been suggested to the author by working logicians. The list of about two hundred people quoted includes eminent logicians, philosophers, scientists, writers and poets, politicians, singers, etc. Naturally, all these people have different views on what logic is and what logicians do. Their variety of approaches made the book at once funny and critical.

Although some quotations are definitely amusing, they have absolutely nothing to do with logic, and the author doesn't explain their connection to logic properly. One can also regret the incomplete referencing of the quotations: some are not referenced at all, while for others the pages are not mentioned. Furthermore, some quotations are unfortunately too long and some others too complex for such a book. Finally an overcharged presentation makes it difficult to use for quick browsing.

Despite theses quibbles, the book is very pleasant. Though it is clearly intended for a large public, it is surely more than a pastime for train trips.

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Oskar Becker und die Philosophie der Mathematik. Neuzeit und Gegenwart, edited by Volker Peckhaus, Wilhelm Fink Verlag, München, 352 pp.

Oskar Becker (1889–1964) was a German mathematician who turned to philosophy after having taken his doctorate in mathematics in 1914. In 1923 Becker submitted his Habilitation on the phenomenological foundations of geometry at the University of Freiburg, and soon became Edmund Husserl's assistant there. A few years later, after the publication of his most important work "Mathematische Existenz" (1927), he was promoted to the position of Extraordinarius. He stayed in Freiburg until leaving for Bonn for a chair in philosophy in 1931. In 1930 Becker published a pioneering modal logic essay "Zur Logik der Modalitäten" and later, in the early 1950s, he also contributed to the early development of deontic modal logic with his *Untersuchungen über den Modalkalkül*, but unlike the other pioneers in this field, Georg Henrik von Wright and Jerzy Kalinowski, he did not elaborate his theory any further.

This anthology, *Oskar Becker und die Philosophie der Mathematik*, is mostly based on a series of Becker colloquia in the University of Hagen during the years 2001–2003 and focuses on Becker's activity in the fields of the history and philosophy of mathematics. Most of the contributions are written in German but some of them are in English. An introduction written by the editor provides the reader with a convenient concise account of Becker's life and accomplishments, and also briefly comments upon the structure and content of the anthology itself (pp. 9–14).

The opening two articles are devoted to the history of mathematics. The first article consists of Eberhard Knobloch's account of Johannes Kepler's and Paul Guldin's search for new insight to heuristic and indirect proof-methods in relation to the work of Archimedes (pp. 15–34). In the second article, Christian Thiel discusses Becker's conception of the notion of mathematical existence in relation to Hieronymus Zeuthen's thesis, according to which geometrical constructions, together with the accompanying proofs, served, in ancient mathematics, fundamentally as existence-proofs for the constructed quantities (pp. 35–45). These historical case studies are followed by Antonello Giugliano's phenomenological survey on the philosophy of time (pp. 47–58), and two essays discussing Becker's philosophy of mathematics: Hans Poser's "Ontologie der Mathematik im Anschluß an Oskar Becker" (pp. 59–77) and Volker Peckhaus's "Impliziert Widerspruchsfreiheit Existenz?" (pp. 79–99). The last mentioned two contributions provide a convenient map of the field of mathematical ontology, a succinct description of David Hilbert's formalist project, and an insightful account of Becker's opposing first-personal phenomenological conception of mathematical existence.

Mark van Atten has prepared two interrelated contributions to this volume. The first one discusses Husserl's, Weyl's, and Becker's phenomenological reception of L. E. J. Brouwer's intuitionist philosophy of mathematics (pp. 101–117), while the second one provides an annotated version of Becker's correspondence with the Dutch intuitionist Arendt Heyting (pp. 119–142). Both of these interesting pieces are based on archival materials and are written in English.

Johannes Emrich's article "Beckers Anwendung der Denkfigur des offenen Horizonts auf mathematische Objekte" (pp. 143–152) relates to the famous early 20th century *Grundlagenstreit* and focuses on Becker's phenomenological conception of the foundations of mathematics and science. This article is followed by another important contribution to the same discussion, namely Paolo Mancosu's and Thomas Ryckman's careful scrutiny of Becker's correspondence with Hermann Weyl during the period 1923–1927 (pp. 153–227). Mancosu and Ryckman first give, in English, an illuminating brief account of Weyl's and Becker's early careers and their relations to the phenomenological movement, they then scrutinize the intellectual interaction of Weyl and Becker in great detail, before finally adding as an appendix, a careful edition of four letters from Becker to Weyl. Unfortunately, Weyl's replies to Becker were lost in the Second World War. This well-written and highly interesting contribution dominates the anthology. It can be regarded the heart of the work.

Mancosu's solo-essay " 'Das Abenteuer der Vernunft': O. Becker and D. Mahnke in the Phenomenological Foundations of the Exact Sciences" (pp. 229–243) offers a convenient introduction to the Becker-Mahnke correspondence in the years 1926–1933. Mancosu's summary is followed by the correspondence itself, edited by Bernd Peter Aust and Jochen Sattler, consisting of eleven letters from Becker to Mahnke and two of Mahnke's replies to Becker (pp. 245–278).

The work also contains two papers discussing set-theoretical issues, that is, Volker Peckhaus's "Becker und Zermelo" (pp. 279–297) and Pirmin Stekeler-Weithofer's "Zu einer prototheoretischen Begründung der klassischen Mengenlehre" (pp. 299–324). Peckhaus's second article nicely complements his first essay in this anthology by discussing Becker's relation to Hilbert's early collaborator Ernst Zermelo who first axiomatized set-theory. The article closes with an edition of Becker's letter to Zermelo on December 31st, 1930 (pp. 291–294). Stekeler-Weithofer's essay completes the picture from the perspective of the foundations of classical set-theory.

Matthias Wille's essay "Dem Unendlichen einen finiten Sinn beilegen" (pp. 325–350) wraps up the anthology. It discusses proof-theory and more precisely the relations between Becker, Gerhard Gentzen, and Paul Lorenzen.

All in all, *Oskar Becker und die Philosophie der Mathematik* is a carefully composed and useful collection of interesting essays, and other source materials, to the history and philosophy of mathematics. It is a welcome contribution to a better understanding of the interaction between philosophers and mathematicians during the Golden Years of foundational studies in the early 20th century. However, it must be said that a Becker bibliography and an index (at least of names) would have greatly facilitated the usability of this work.

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MICHIEL VAN LAMBALGEN and FRITZ HAMM. ***The proper treatment of events.*** Explorations in Semantics, no. 4. Blackwell Publishing, Oxford, 2005, xii + 251 pp.

One of the differences between formal languages and natural languages is the fact that in natural languages predicates do not seem to have a fixed arity. Adjuncts and modifiers can be added seemingly in any number. This is impossible in predicate logic. In *Brutus kills Caesar violently with his knife* both *violently* and *with his knife* are syntactically not required. But since they can be added, we must decide how many arguments the predicate kill actually takes. If we assume it is a 4-place predicate we must always supply four arguments, while if it were a 2-place predicate we would not know where the two extra arguments should go. Events provide a solution to this problem. Rather than saying that the sentence means kill(brutus, caesar, knife, violently) we shall now say that it asserts the existence of an event of killing whose actor is Brutus, whose patient is Caesar, whose manner was violent, and whose instrument was a knife. Thus arguments and adjuncts are analysed on a par, and they are all translated as adjuncts in the formal language (see Terence Parsons, ***Events in the Semantics of English. A Study in Subatomic Semantics***, Current Studies in Linguistics, no. 19, MIT Press, 1994).

The introduction of events raised at least as many questions as it answered. Their ontological status was and still is unclear. For example, if John is running between 2 and 4, how many events of running do we have? One, or infinitely many? For example, do we also have John's running between 2:30 and 3:00 and John's running between 2:15 and 3:35? This question is answered differently by different people, if at all. The literature is full of proposals that analyse the meaning of simple sentences using an infinite series of events rather than a single one. However, recall the dictum 'No entity without identity'. If we are not able to say what constitutes an event there is no theory of events whatsoever and the formalisation using events remains a mindless game: it cannot be grounded in reality.

The linguistic literature passes this state of affairs mostly with silence. However, this threatens to undermine the usefulness of events. Truth conditions are given in models that have events in them but we have no indication as to how to evaluate sentences in real life. For we are not given any guideline as to how to find events in the real world. True, one may have similar problems with the basic notions such as objects themselves; but there is a consensus that the notion of an object inasfar as its extensional side is concerned is subject to the laws of physics, among other. This means that we can determine with some certitude where an object is located at a given time.

This is not a necessary state of affairs: what we need to ask for is how we can identify events in reality and what governs their behaviour. This is what the present book is about. Its aim is nothing more and nothing less than what its title suggests: the proper treatment of events. To do that it draws on insights from physics, computer science and robotics. The reason for this exceptional synthesis is that all of these disciplines actually have looked at the problem of *change*. What they converge on is this: in order to study events one must give up the idea that motion is nothing but a succession of snapshots. It is not the place here to subject this problem area to scrutiny; suffice it to say that the idea of continuity is meaningless if that were so. Rather, one should embrace the idea that in inertial motion there is no real change. Inertial motion is the natural flow of events (*sic!*) unless something intervenes. The law of inertia is a physical law. It has given physicists headaches, too, see Carl Friedrich von Weizsäcker, ***Zeit und Wissen***, Carl Hanser Verlag, 1992, but it remains a fact of life, e.g., that this billiard ball will bounce if nothing unforeseen intervenes. The second ingredient is foreshadowed in the word 'unforeseen'. Of course, the billiard ball might not bounce—for various reasons. The fact that it is unforeseen depends (if you believe in determinacy) simply on our ignorance of the exact state of the world; but that does not eliminate our problem of saying what the word 'unforeseen' is doing here. Moreover, that we do not know everything is simply a fact and has to be taken into account when modelling reasoning and semantics. And so we are thrown into the arms of logic programming: we say goodbye to omniscience and make do with our limited knowledge.

If we would do only physics, however, we would not get very far. For language talks about a lot more than that. However, it does seem that constant change is somehow hardwired as 'no change'. If cars are driving past this is no reason to be alert. But if a car is changing gear or starts to turn it gets our attention. The idea that constant change actually means 'no change' has been taken seriously in robotics. Reasoning in presence of constant change is what is required. This has given rise to the introduction of *fluents*. Fluents may be constructed as time dependent properties or quantities, but they are seen here as simple entities. There is a fluent of the door being open, a fluent of Max crossing the road, a fluent of Columbo building a house, and so on. Fluents are of course interdependent. There are some basic predicates, for example clipped or terminates, that describe the way events and fluents interact with each other. What we get is a many-sorted first-order logic that comprises the following: objects, real numbers, variable quantities such as states and activities, spatial locations, event types that mark the beginning and end of time dependent properties. All this is needed to describe a simple affair such as Max crossing the road and the imperfective paradox.

It may sound mysterious why we have events in conjunction with fluents. However, the authors remind us right at the start of the book of a known problem of physics: that of defining time. What is time and how is it we can witness the progress of time? The answer is that time is actually constructed from states-of-affairs. If this door is open and closed on different occasions, this means that time must have passed in between those two states. Time has no independent reality (nor does space). Time is intimately connected with causality. It is the causal structure of the world that lets us construct time from mere states-of-affairs. It turns out that this is exactly the way in which humans deal with the world. We construct time from what happens around us. In the present context this means: events are defined through changes in the truth of fluents. If Max is reaching the top of Ben Nevis then that is an event; as such it terminates Max's climbing Ben Nevis. Notice that climbing Ben Nevis is an activity, and requires among other gradual change in height.

Now we have all the ingredients together: event calculus, logic programming, and some physics. This is what the first part of the book is about. It defines the event calculus, which is a logical theory. Then it turns to logic programming and reasoning with time and events. In the second part the authors put this theory to use. They propose an analysis of aktionsart on the basis of the event calculus. Then they give an in-depth account of tense in natural language, focussing on some specific cases, such as the passé simple and the imparfait of French. Next they deal with aspect, wth coercion, and finally with event nominalisation.

This book presents a highly innovative approach to the semantics of natural language. The authors manage with admirable ease to draw together insights from different fields and show how their theory can actually explain facts rather than simply assuming them. This is not a trivial achievement: to derive even the most simple sounding conclusion requires a lot of effort. This book is a truly intellectual book, written with love for the subject. I consider it a must for everyone who is interested in events or natural language semantics in general.

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ROBERT S. WOLF. ***A tour through mathematical logic,*** The Carus Mathematical Monographs, Number 30. The Mathematical Association of America, Washington, D.C., 2005, xv + 397 pp.

The Carus Mathematical Monographs "are intended for the wide circle of thoughtful people familiar with basic graduate or advanced undergraduate mathematics encountered in the study of mathematics itself or in the context of related disciplines who wish to extend their knowledge without prolonged and critical study of the mathematical journals and treatises." Robert Wolf provides a "tour" of mathematical logic for non-logicians, be they students or professionals. While Herstein could write a monograph (Number 15) on *Noncommutative Rings* and expect his readers to know some algebra, Wolf knows that in writing about logic one must start at the very beginning. As a consequence, the core of this book—the first five chapters—reads like a first course in mathematical logic but without most of the proofs and with a very selected set of topics. These five chapters cover predicate logic, basic axiomatic set theory (through ordinals and cardinals), computability theory (through the basic properties of recursively enumerable sets), the incompleteness theorems, and model theory (through some of the first results on saturated models and on real closed fields). The final three chapters address three more advanced topics that the general mathematical audience might find especially attractive.

Several features of the book contribute to the success of the first five chapters as an introduction to basic logic for nonlogicians. First, Wolf adopts an informal, conversational style that indeed contributes to the feeling that this is a tour of highlights rather than a survey of results. Wolf's is a fluid, unpretentious voice and he doesn't let the exposition get bogged down in a long sequence of Definitions, Theorems, Proofs and Exercises. It's the right tone for a tour. Second, Wolf does not hesitate to detour down lesser-trodden lanes that might be of special interest to non-logicians. For example, the chapter on computability contains a short but clear discussion of computational complexity including the P = NP problem and the chapter on Gödel's Theorem includes a statement of the Paris-Harrington Theorem. (However the discussion of complexity theory would have been improved had Wolf not decided to omit in this section any attribution of results. For example, the recent result of Agarwal, Kayal and Saxena that the set of primes is decidable in polynomial time is included without any mention of these mathematicians.) Third, whether intentional or not, a unifying theme of logic as the study of definability emerges. For example, I was skeptical at first by what I thought might be an overemphasis on preservation theorems in model theory (e.g., a theory is preserved under union of chains just in case it is equivalent to a Π_2 set of sentences). But on reflection, I saw this fitting into a clear, recurring theme of analyzing the complexity of description of mathematical objects. Of these five chapters, the nonlogician might very well find the chapter on incompleteness theorems most interesting. That chapter is particularly successful at covering the territory without getting bogged down in technical details as expositions of Gödel's Theorem often do.

While the first five chapters form the core of this book, it's the last three chapters (Contemporary Set Theory, Nonstandard Analysis, and Constructive Mathematics) that might sell it. As luck would have it, I received my review copy on the day that I was beginning to prepare a talk on the continuum hypothesis for a general mathematical audience. I expected many in my audience to at least vaguely recall that CH is independent of the usual axioms for set theory but that almost no one would know that the years since Cohen have seen a remarkable growth in our understanding of sets of reals. Wolf's chapter on contemporary set theory told me almost everything that I needed to know to prepare my talk. This is a gorgeous chapter that introduces the reader to forcing and large cardinals and then carefully tells the the story of modern descriptive set theory ending in a description of Woodin's program for deciding (or at least shedding light on) the continuum problem. While the chapter is demanding, I think a patient reader with a reasonable familiarity with elementary topological and measure-theoretic properties of sets of reals could follow much of this discussion and would appreciate both the difficulty involved in understanding the structure of arbitrary sets of reals but also the progress made in this regard in descriptive set theory over the past thirty years. It is here that the theme of definability has its payoff. The last two chapters concern nonstandard analysis and constructive mathematics. Again, these are quite attractive topics for the general mathematician and the treatment that Wolf gives them is engaging.

Wolf covers a lot of ground in this tour and I suppose that most everyone will disagree with some of his choices as to what to include. I myself found the chapter on computability theory disappointing. Unlike set theory, which Wolf presents as a vital, contemporary field of study, the treatment of computability is very brief and ends with Rice's Theorem. Since so much contemporary computability theory is about relative computability and Wolf does take the time to introduce oracle Turing machines (in the section on complexity theory), a few pages on the Turing degrees could have given a more accurate picture of what computability theorists are interested in. Model theorists or philosophers of mathematics might have their own disappointments. Given his page limit however it really is difficult to fault Wolf for his choices and I would be pleased if more mathematicians were familiar with the mathematics in this book.

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