MODAL OPERATOR THEORY AND FORMAL EPISTEMOLOGY

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AGENDA

- Epistemological Status
- Modal Operator Theory
- Modal Operator Theory – Formal Details

STATUS

Contemporary epistemological studies are roughly either carried out in a

(1) **mainstream** and traditionally rather informal way using largely common-sense considerations and concentrating on sometimes folksy and sometimes exorbitantly speculative examples / counter-examples, or

(2) **formal** way by applying a variety of tools and methods from logic, computability theory or probability theory to the theory of knowledge

STATUS

Like the blind men around the elephant, philosophers seize the concept of knowledge only by its various parts

Mainstream epistemology seeks necessary and sufficient conditions for the possession of knowledge

Formal approaches to epistemology either proceed axiomatically or concentrate on learning and knowledge acquistion issues

The traditions have proceeded
largely in isolation from one another

STATUS

Learning theorists do not formalize knowledge as a modal operator,

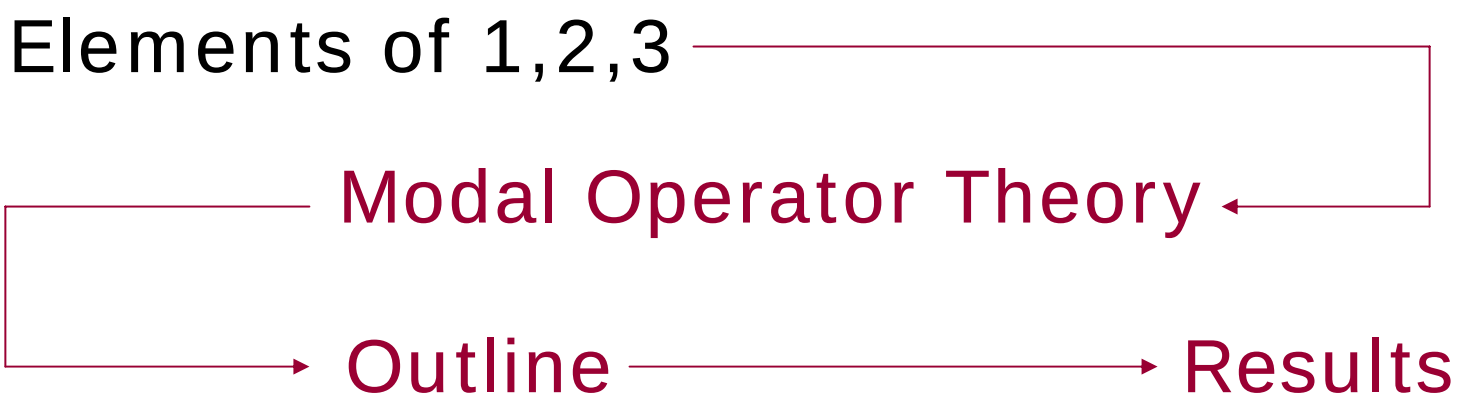
modal logicians have only recently begun to care about intuitive interpretations of their calculi,

and those who would find intuitive definitions of knowledge focus on narrowly folksy examples rather than on scientific method

Such is the fragmented and
distributed state of epistemology today

OUTLINE

- 1. Mainstream Epistemology
- 2. Epistemic Logic / Modal Logic
- 3. Computational Epistemology



MAINSTREAM EPISTEMOLOGY

Mainstream epistemology seeks necessary and sufficient conditions for the possession of knowledge

The focus is on folksy examples and counterexamples in which reasons undercut reasons that undercut reasons

Why?

Getting Gettier
(...and other recalcitrant ***skeptical*** problems)

MAINSTREAM EPISTEMOLOGY

FORCING – Nozick, Dretske, Lewis, (Goldman)

Justification

Forcing

Externalism

Internalism

Reliabilism

Whenever knowledge claims are challenged by alleged possibilities of error, the strategy is to show that the possibilities of error fail to be genuine in the relevant sense

Examples

Avoiding error

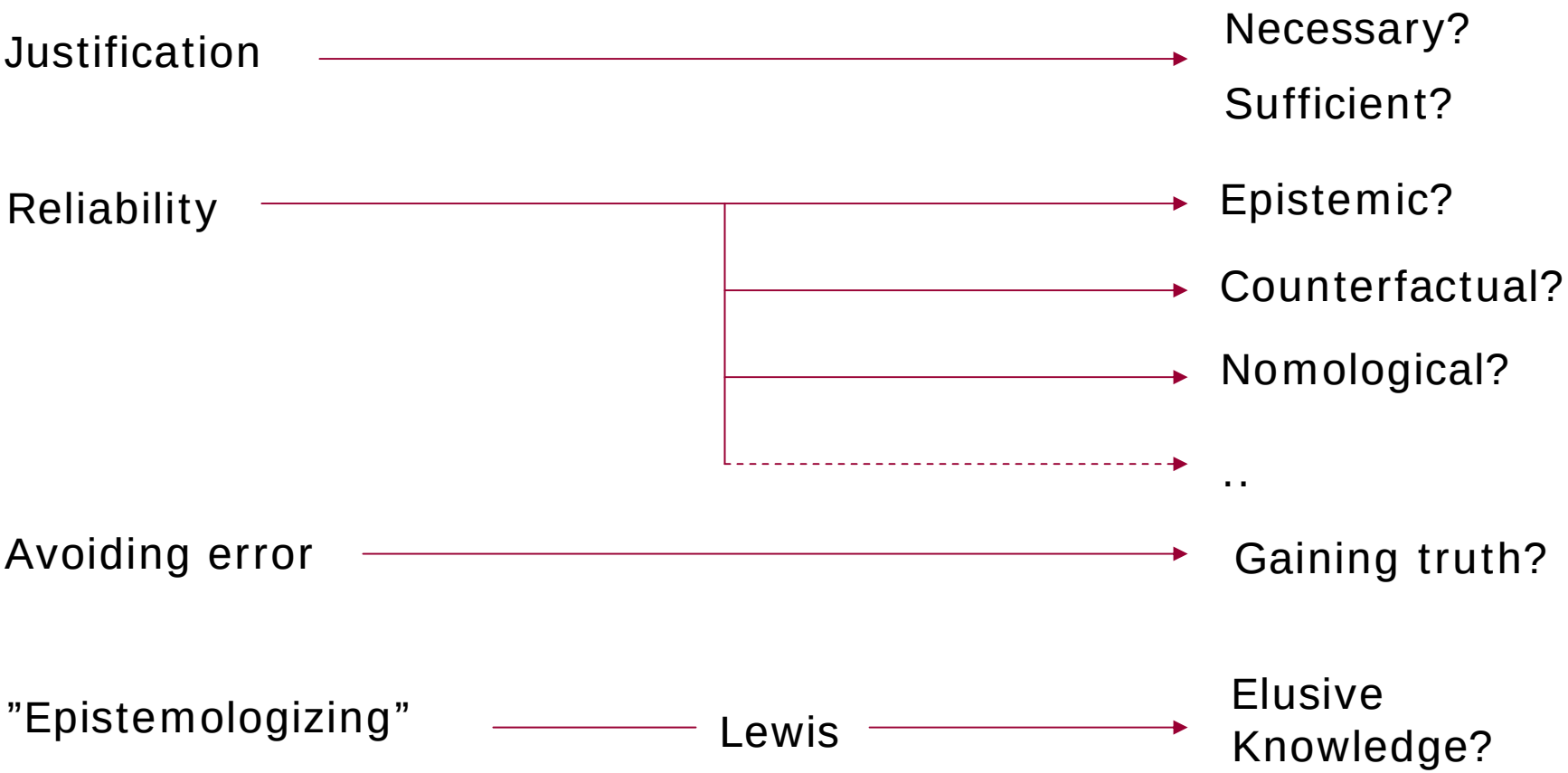
Counterexamples

Gaining truth

Skepticism

MAINSTREAM EPISTEMOLOGY

PROBLEMS



MAINSTREAM EPISTEMOLOGY

1 **person perspective**. A perspective on scientific inquiry is 1st person if it is considered what an agent can solve, can do or defend considering the available means for an end given the epistemic environment he is sunk into

3 **person perspective**. A perspective on scientific inquiry is 3rd person if it is considered what an agent could solve, could do or defend considering the best means for an end independently of the epistemic environment he is sunk into

EPISTEMIC LOGIC

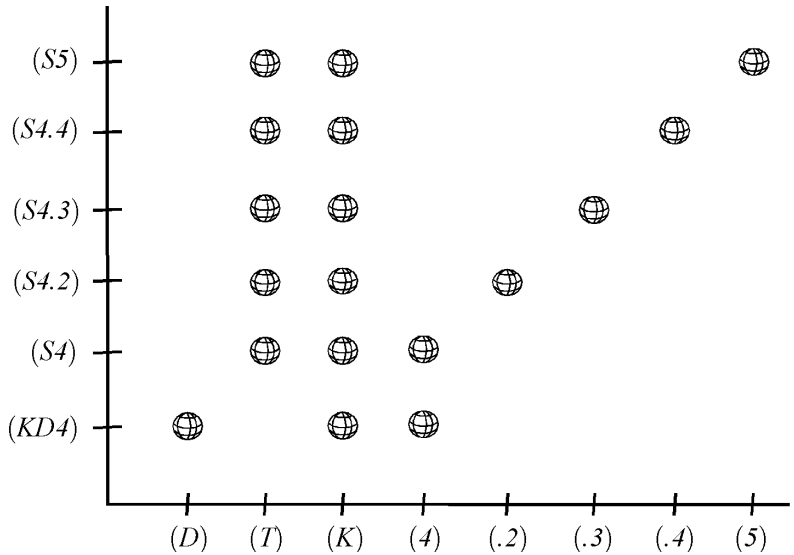
Epistemic logic, proceeds axiomatically. ”An agent knows that *A*” is formalized as a modal operator in a formal language which is interpreted using the standard apparatus of modal logic

The hope is that cataloguing the possible complete systems of such logics will for a picking of the most appropriate or intuitive ones (~**S4** – **S5**)

EPISTEMIC LOGIC

- EPISTEMIC AXIOMS

- (**D**) *Consistency*
- (**T**) *Truth*
- (**K**) *Cogency*
- (**4**) *KK-thesis*
  - ..
- (**5**) *Wisdom*



Epistemic axioms: D – S5

Strength of epistemic operator in terms of validity

EPISTEMIC LOGIC

ISSUES

Operators

Axioms

Epistemic

Forcing

Systems

Epistemic logic delimits the set of possible worlds relevant for the epistemic operator by letting the accessibility relation between the possible worlds comply with certain classical relational properties

Soundness

Syntax

PROP

Semantics

Completeness

Possible worlds

Accessibility

EPISTEMIC LOGIC -> AXIOMS

Truth: $K_\delta A \rightarrow A$ (JTB – knowledge (1)

Cogency: $K_\delta(A \rightarrow B) \rightarrow K_\delta A \rightarrow K_\delta B$ (Closure)

Pos. intro: $K_\delta A \rightarrow K_\delta K_\delta A$ (KK-thesis)

Pos. intro: $\neg K_\delta A \rightarrow K_\delta \neg K_\delta A$ (Wisdom)

Moore: $B_\delta A \rightarrow B_\delta K_\delta A$ (Certainty)

Modal epistemic systems furnish various responses to skepticism

EPISTEMIC LOGIC

Should epistemic logic hook up with broader epistemo-logical issues?

> The search for the correct analysis of knowledge, while certainly of extreme importance and interest to epistemology, seems not significantly to affect the object of epistemic logic, the question of validity of certain epistemic-logical principles. [Lenzen 1978]

The relation between epistemic logic and mainstream epistemological issues is dim

EPISTEMIC LOGIC

Epistemic logic begins as a study of the logical behavior of the expression of the form '*b* knows that.'' One of the main aims of this study is to be able to analyze other constructions in terms of 'knows''by means of '*b* knows that'.' The basic notation will be expressed in the notation used here by K_b.' This symbolization is slightly misleading in that a formula of the form K_bS the term *b* for the agent (knower) is intended to be **outside** the scope of *K*, not inside as our notation might suggest. [Hintikka 1998]

EPISTEMIC LOGIC

Epistemic-logical principles or axioms are relative to an agent or method which may or may not validate these principles

The primary role of the methods in classical epistemic logic is to index the accessibility relation

Indices on accessibility relations will not suffice for epistemological pertinence simply because there is nothing particularly epistemic about being indices – we want to know ***how the agent has to behave in order to obtain knowledge***

An Inactive Agent

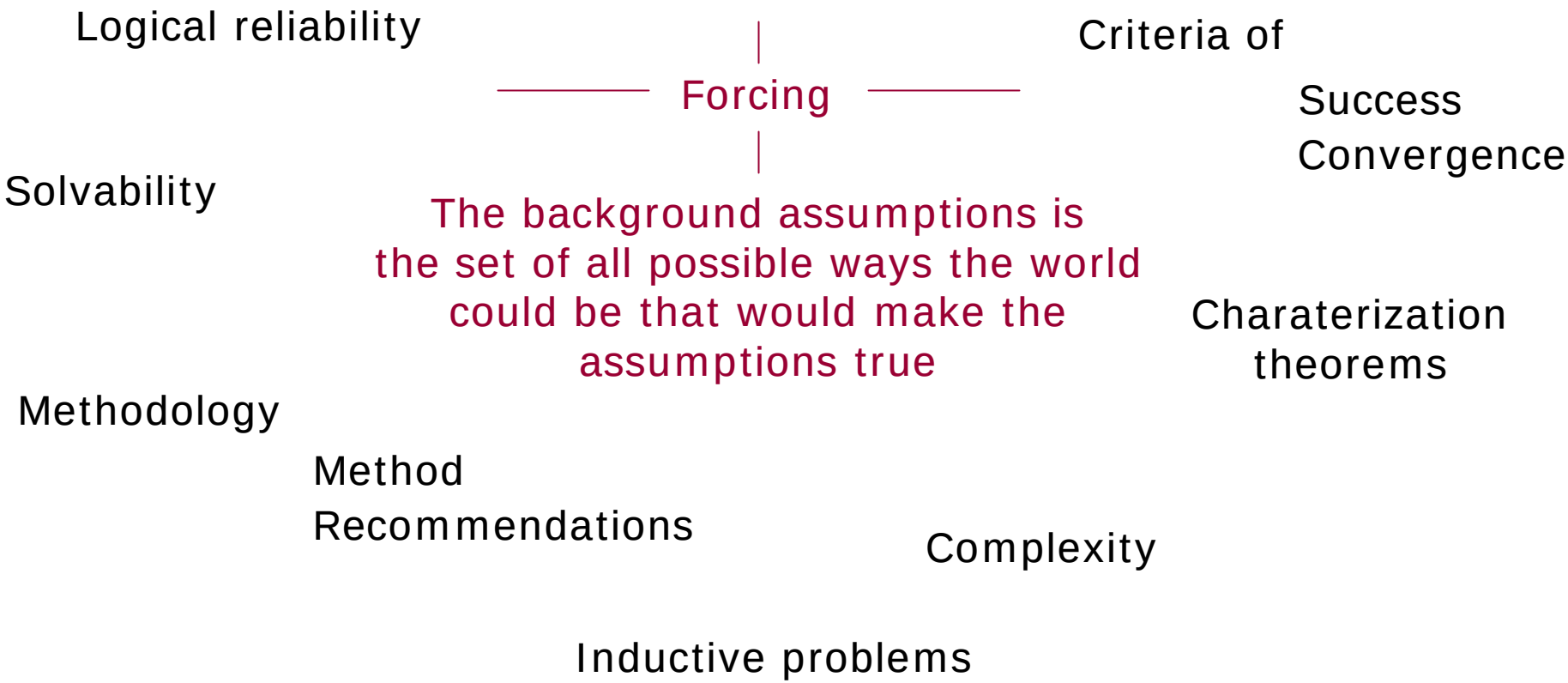
COMPUTATIONAL EPISTEMOLOGY

Computational epistemology, also known as formal learning theory uses logical and computational techniques to study when guaranteed convergence to a correct answer is feasible

Computational epistemology is the formal study of inductive problems and their intrinsic solvability for both ideal and computational agents

COMPUTATIONAL EPISTEMOLOGY

ISSUES



COMPUTATIONAL EPISTEMOLOGY

Methodological recommendations imposed on inquiry methods may either be:

- ***Permissive*** in the sense that a method entertaining the recommendation is guaranteed to arrive at the correct answer for some suitable convergence criterion

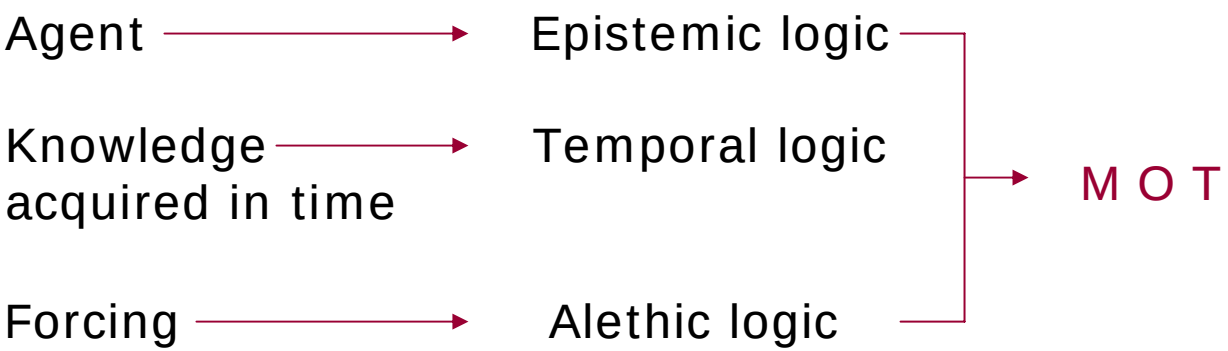
- ***Restrictive*** in the sense that a method entertaining the recommendation is demonstratively barred from arriving at the correct answer for some or all convergence criteria

? If rational scientific inquiry is characterized by convergence to, or approximate convergence to, the truth, then why impose recommendations which destines the inquiry method to fail

! Whenever methodological recommendations are advanced in the aim of finding the correct answer, check whether the canons have the intended truth-conducive qualities

MODAL OPERATOR THEORY I

Modal Operator Theory (MOT) denotes the cocktail obtained by mixing epistemic, alethic, tense logic with a few concepts from computational epistemology in order to study the strength and validity of limiting convergent knowledge



MODAL OPERATOR THEORY I

THESIS

Knowledge may be characterized by convergence to a correct hypothesis in the limit of empirical scientific inquiry.

The primary aim is not to say whether convergence will or will not occur

It is rather to systematically investigate the proposal that such convergence, if it occurs, is descriptive of scientific knowledge from a logical point of view

MODAL OPERATOR THEORY I

ISSUES

Reliability

Limiting convergence

Forcing

Possible worlds

The background knowledge consists of all the possible worlds evidentially consistent with the actual world observed until 'now'

1 vs 3 person

Epistemic operators

Validity of knowledge

Methodology

Method Recommendations

Synchronicity Diachronicity

MODAL OPERATOR THEORY I

ELEMENTS OF SCIENTIFIC INQUIRY

- A space of possible worlds
- A formal concept of background knowledge
- A space of entertainable hypotheses
- A definition of correctness of a hypothesis in a possible world
- A method of scientific discovery
- A definition of limiting convergence
- A definition of limiting convergent knowledge
- A set of methodological recommendations

MODAL OPERATOR THEORY I

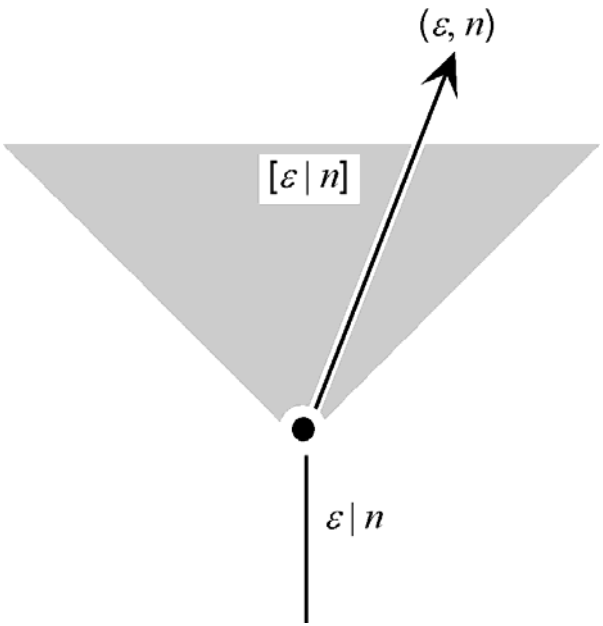
DEFINITION – evidence stream

An evidence stream ε is an ω–sequence of natural numbers, i. e. $\varepsilon \in \omega^{\omega}$

DEFINITION – possible world

A possible world is a pair consising of an evidence stream ε and a state coordinate n, i. e. (ε, n) where $\varepsilon \in \omega^{\omega}$ and $n \in \omega$

MODAL OPERATOR THEORY I



1. $(\varepsilon, n) \vDash Fh$ *iff* $\exists k > n : (\varepsilon, k) \vDash h.$
2. $(\varepsilon, n) \vDash Gh$ *iff* $\forall k > n : (\varepsilon, k) \vDash h.$
3. $(\varepsilon, n) \vDash Ph$ *iff* $\exists k < n : (\varepsilon, k) \vDash h.$
4. $(\varepsilon, n) \vDash Hh$ *iff* $\forall k < n : (\varepsilon, k) \vDash h.$
5. $(\varepsilon, n) \vDash \Box h$ *iff* $\forall (\tau, m) \in \mathcal{W} : (\tau, m) \vDash h.$
6. $(\varepsilon, n) \vDash \boxdot h$ *iff* $\forall k \in \omega : (\varepsilon, k) \vDash h.$
7. $(\varepsilon, n) \vDash \boxminus h$ *iff* $\forall (\tau, m) \in [\varepsilon \mid n] : (\tau, m) \vDash h.$

Handle = observed evidence

Fan = background knowledge

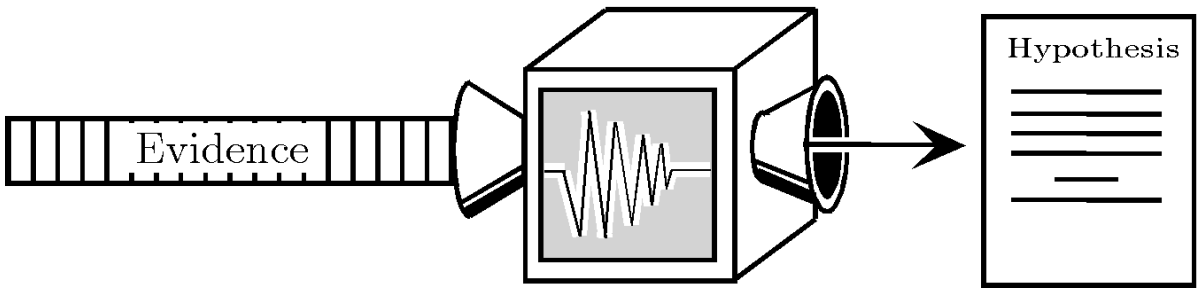
Hypotheses = sets of possble worlds

Truth = identification of actual world forever after

MODAL OPERATOR THEORY I

DEFINITION – discovery method

A discovery method δ is a function from finite initial segments of evidence to hypothesis



METHODOLOGICAL RECOMMENDATIONS - discovery

- Consistency
- Perfect memory
- Consistent expectation
- Infallibility

MODAL OPERATOR THEORY I

DEFINITION – limiting convergence for discovery

δ discovers hypothesis h in the limit in (ε, n) iff there is a time k s. t. for all $n' \geq k$ in all worlds in the background knowledge: δ conjectures h at n'.

DEFINITION – limiting convergent knowledge

δ may know hypothesis h in the limit, iff there exists a possible world which validates δ's knowledge of h, i.e.

1. h is correct.

2. δ conjectures h after some finite evidence sequence has been read and continues to conjecture h in all future.

MODAL OPERATOR THEORY I

Observe that since the agent is a function from finite initial segments of evidence to hypotheses there is no knowledge without the method conjecturing a hypothesis

Thus, the agent is *actively within the scope of the operator* so $K_\delta h$ in modal operator theory is genuinely faithful to the intended meaning of the standard formalization of the knowledge operator

MODAL OPERATOR THEORY I

The set-theoretical characterization allows for a formalization in a modal propositional calculus

We can then ask the following pair of questions:

Which epistemic axioms can be validated by an epistemic operator based on the definition of limiting convergent knowledge

Does the validity of the various epistemic axioms relative to the method depend upon enforcing methodological recommendations

MODAL OPERATOR THEORY I

THEOREM

If knowledge is defined as limiting convergence, then knowledge validates S4 if the discovery method has consistent expectations

Axiom (T) does not require enforcing methodology for validity

Axiom (K) does not require enforcing methodology for validity

Axiom (4) (the KK-thesis) does require enforcing methodology for validity!

MODAL OPERATOR THEORY I

A methodological recommendation may according to MOT be either:

Boosting **in the sense that the** methodological recommendation is conducive to validating epistemic axioms and systems

Debilitative **in the sense that the** methodological recommendation is an impediment to validating epistemic axioms and systems

Neutral if it is **neither** boosting or debilitative..

Classically it has been argued since James that the KK-thesis and limiting convergence are incompatible:

> This does not entail that Θ knows he knows the answer, since Θ may lack any reason to believe that his hypotheses have begun to converge. [Martin & Osherson 98]

How to have the cake and eat it too

DEFINITION – synchronicity

An epistemic or doxastic axiom is synchronic if the consequent obtains by the very same time the antecedent obtains

DEFINITION – diachronicity

An epistemic or doxastic axiom is diachronic if the consequent either obtains later or would have obtained later than the antecedent even if things had been otherwise

From the MOT-perspective it is possible to validate the KK-thesis when recognizing:

- That the KK-thesis can be a diachronic axiom from the third person perspective
- That the method has to converge to the fact that it knows, then wait around and then later converge to the fact that it knows that it knows *h*
- That only *consistent expectation* allows for this kind of strategic forcing relative to the background knowledge

MODAL OPERATOR THEORY II

See "Modal Operator Theory - Details" slides ...

CONCLUSION AND PERSPECTIVES

The point of forcing is to square away confusion and misunderstandings between mainstream and formal epistemologies

A mainstream epistemological paradigm may be a forcing strategy, and a formal one may be too – the question is then how they do force respectively putting them on par for camparison

The point of distinguishing between first and third persons perspectives on inquiry is to square away confusion and misunderstandings between mainstream and formal epistemologies

For example, in criticising some position, whether formal or informal, without noticing that the criticism is based on a third person perspective and the position advocated is first person may turn out to be criticising an apple for not being an orange

FROM JUSTTICATION TO METHODLOGY

The justification condition of epistemology ends up in methodology as the study of how science arrives at its posited truths, i.e. how beliefs are justified by the canons, norms or recommendations and internal workings of the method applied or agent in question

'For if use of method conduces to truth, then, given the relation between method and justification, the warrant provided by the method is warrant with respect to truth' [Sankey 99]

FROM JUSTTICATION TO METHODLOGY, CONT

Methodological recommendations, truth-conduciveness, reliability, convergence, strategies for winning games, changing your beliefs economically and reliably and the like are at the very **core** of many formal epistemological proposals.

In general what the mainstream epistemologists are looking for seems to be what the formal epistemologists have to offer

TAXONOMY OF RELIABILITY

		1	2	3	4	5	6	7	8
Goldman			X	X			X	X	
Nozick		X		X			X	X	
Lewis		X		X			X	X	
Hintikka		X			X	X		X	
Kelly		X			X	X		X	X
Hendricks		X			X	X		X	X
...									

1. Categorical reliability
2. Stochastic reliability
3. World-dependent
4. World-independent
5. Apriori reliable
6. Aposteriori reliable
7. Certainty convergence
8. Limiting convergene

ACTIVE AGENTS

'First Generation Epistemic Logic'

'Second Generation Epistemic Logic'

- Active Agents: Game-theory, belief revision theory, probability theory, computational epistemology .. Active agents within the scope of the operators

EPISTEMOLOGY AXIOMATIZED

		CE	COE	LE	MOE
N:	$\frac{A}{K_{\Xi}A}$	1^s	$1^s/3^s$	$1^s/3^s$	3^s
K:	$K_{\Xi}(A \rightarrow A') \rightarrow (K_{\Xi}A \rightarrow K_{\Xi}A')$		$(1^s)/(3^s)$	$1^s/3^s$	3^s
T:	$K_{\Xi}A \rightarrow A$	1^s	$1^s/3^s$	$1^s/3^s$	3^s
4:	$K_{\Xi}A \rightarrow K_{\Xi}K_{\Xi}A$		1^s	$1^s/3^s$	3^d
5:	$\neg K_{\Xi}A \rightarrow K_{\Xi}\neg K_{\Xi}A$		(1^s)	3^s	

TABLE 5 CE: Counterfactual Epistemology, COE: Contextual Epistemology, LE: Logical Epistemology, MOE: Modal Operator Epistemology. 1: First person perspective, 3: Third person perspective. s: synchronicity, d: diachronicity. (,): context-sensitive validity