Contemporary epistemology is divided into two main traditions with almost no interaction: (a) mainstream epistemology which pursues two goals: providing necessary and sufficient conditions of knowledge, defeating the skeptics’ endeavour to undercut claims to knowledge, (b) formal epistemology which brings results of epistemic logic, game-theory, computability theory and learning theory to bear on epistemology. VFH’s very original book fills an important gap in the literature. Not only does it bring these two trends together, taking into account the latest developments in the field, it also states new results, due to the author, which are of major importance such as the incompatibility of perfect memory with consistent expectation.

A unifying theme underlies mainstream and formal philosophy, namely epistemological forcing. The idea of epistemological forcing is as follows: whenever skeptics cite possibilities of error as arguments against knowledge claims, the strategy is to show that, although they are possibilities of error, they fail to be relevant possibilities of error (p.2). For instance the possible world in which we are brains in a vat filled with nutritious fluids by a malicious scientist who strives to fool us systematically is judged to be too speculative to deserve consideration. What criteria of relevance should we adopt? VFH comes to grips with the question. He examines several answers both within mainstream epistemology and within formal epistemology before putting forward his own answer.

Not all forms of skepticism call for the same kind of response. Borrowing the useful distinction between global and local underdetermination from Kelly, VFH shows that the strategy needed to remove the doubts instilled by the skeptic who troubles us with Cartesian demons is different from that which is needed to cope with Humean doubts concerning induction.

In mainstream epistemology, Nozick’s policy illustrates the first strategy. He appeals to the semantics of subjunctive conditionals to discard possibilities entertained by the skeptic: for knowledge possession, one does not have to consult all possible worlds, as the skeptic would insist. It is enough that the consequent $B$ holds in those possible worlds close to the actual worlds such that the antecedent $A$ holds.

For all its power against extravagant reasons of doubting, Nozick’s strategy proves inefficient when used to appease the down-to-earth doubts that are raised against induction. Yet, tying Nozick’s epistemological considerations to Kelly’ computational epistemology,
VFH shows that sticking to convergence with certainty in close worlds, as the counterfactual proposal usually does, may also just be another way of using forcing to evade the problem of induction (p.58). This unexpected combination of counterfactual epistemology and computational epistemology is a perfect illustration of the kind of interaction between mainstream and formal epistemology which VFH is tracking throughout his book.

VFH does not rest content with commenting other epistemologists’ ways of fighting the challenge of skepticism. He also makes a very innovative contribution of his own to the definition of a criterion of elimination of irrelevant possible worlds. To bring that out I have to open a quick parenthesis and recall VFH’s new account of possible worlds adopted from Kelly’s *The logic of Reliable Inquiry*.

In Kelly’s set up, a possible world is a pair consisting of an evidence stream ε and a state coordinate n. A finite initial segment of an evidence stream is called ‘handle’. The set of all infinite streams that extend the handle is called ‘fan’. The possible worlds outside the fan are inconsistent with the evidence. They should be dismissed as irrelevant by the inquirer. Yet uncountably many evidence streams remain in the fan. Hence the criterion of relevance just given is a very weak criterion indeed.

Could we design a sieve which, on a purely epistemological basis, would rule out a great deal of the possible worlds in the fan and provide us with a stronger criterion of relevance? Peirce is known to have thought that science in the long run may converge to the truth without ever producing an unequivocal sign of convergence. Inspired by Peirce’s insight, VFH has defined a bunch of new concepts based on the primitive idea of convergence. The concept of successful limiting convergence of a hypothesis h in a possible world (ε, n) is one of them. It means that ‘there is a time k such that for each later n’ in all worlds admitted by the world fan [ε,n], the discovery method δ conjecture h’(p.138).

The notion of successful limiting convergence might be used to carve out a subset of the fan, i.e. the set of the evidence streams which conjecture a given hypothesis h after a finite time and which continue to do so in all future. In this manner a strong criterion of relevance can be obtained (under the proviso that we postulate ‘uniform convergence moduli’ over the relevant possibilities).

Exploiting the apparatus I have just outlined, VFH has worked out an illuminating model for the interpretation of the central notions of epistemology. On the top of that model he has developed a formal language and an axiomatic system powerful enough to express new epistemological concepts. The emphasis is put on the dynamics of knowledge. The knowing
subject ceases to be the bearer of a mental state. From now on he or she becomes an active agent who can learn or teach, discover hypotheses and assess them.

It is possible to define different concepts of convergence. If we adopt a very strong one according to which the discovery method $\delta$ is infallible, the Gettier paradoxes are solved immediately: there is no way the evidence for $h$ can be undercut when the evidence simply entails $h$ (p.140). Here VFH’s formal analysis of knowledge offers new solutions to paradoxes on which Mainstream epistemologists have fumbled for decades.

A very fruitful use of the concept of convergence should be mentioned here. Looking at the standard axioms of epistemic logic from the standpoint of the definition of knowledge as limiting convergence, VFH raises this fundamental question: ‘Which epistemic axioms may be validated by an operator based on the definition of limiting convergent knowledge for discovery methods?’ (p.141).

As expected the rule of necessitation and axiom K are immediately valid. Axiom T is valid too but as soon as agent indices associated with epistemic operators cease to be seen as mere indices and are viewed as arguments of discovery functions instead, the significance of T changes drastically. It no longer ascribes the passive property of truth to knowledge but the active property of resistance to objections, i.e. undefeasibility.

As far as the axiom 4 (positive introspection) is concerned, at first sight it looks invalid if we adopt the Peircean conception of knowledge. As VFH stresses, ‘limiting convergence emerges as the agent is free to oscillate regarding his conjecture, even if he does not know, nor is required to say, when stabilization has occurred’ (pp.26-27). Yet, if we go deeper into the matter and bring metalogical results obtained by the author to bear on the question, it turns out that a qualified version of axiom 4 is valid after all: ‘if knowledge is defined as limiting convergence, then knowledge validates S4 iff the discovery method has consistent expectations’ (p.141).

As to axiom 5 (negative introspection), the convergence definition of knowledge turns it into a patent falsity. If knowledge is defined by convergence then it contrapositively follows that if you have not converged you do not know and, in particular, it follows that you do not know that you have converged.

The dichotomy between first and third person brings to the fore another facet of the complex relationship between Mainstream and Formal Philosophy. In Mainstream philosophy, the special status of the first person is recognized. Moore’s paradox highlights this contrast: the agent pragmatically contradicts himself or herself in saying ‘$\varphi$ but $I$ do not
believe \( \varphi \)’ . If we switch to the third person or to the past tense the paradox disappears: neither ‘\( \varphi \) but he or she does not believe \( \varphi \)’ nor ‘\( \varphi \) but I did not believe \( \varphi \)’ are paradoxical \(( \text{Gochet 2004}(\text{Gochet & Gribomont 2006, pp.134-135}), \text{(Gochet 2004,121-123)} \).

Among formal epistemologies computational epistemology restores the balance and takes up a third person approach from the start. Here are questions belonging to third person epistemology: ‘Is there an intrinsic characterization of solvability of the epistemic problem in terms of topological complexity?’ A question of that kind is currently raised within the framework of complexity theory. As VFH stresses, ‘these questions are asked largely independently of the epistemic environment that some agent is sunk into and arise from the third-person perspective of epistemology (p.128)’.

The appeal to computational epistemology produces a rich harvest of new insights. Just to give an example, it brings out that epistemological problems which withstand classical treatments may have reliable solutions if we adopt *successful convergence criteria* that are weaker than halting decision. If the criteria of convergence are weakened the watertight separation between *discovery methods* and *assessment methods* collapse and the distinction between induction and deduction turns to be a difference in degree rather than a difference in kind.

The principle ‘every event has a cause’ was given a very different *status* in Kant’s *Kritik der reinen Vernunft* and in Russell’s *Human Knowledge, Its Scope and Limits*. To put it bluntly, Kant ascribes it the status of a component of the *structure of the human understanding* which gives shape to empirical data. On the contrary Russell ranks it among the *postulates of scientific inference*.

Far from being an outdated alternative, these two philosophical stances have inspired the following epistemological question which VFH raises about the axioms of epistemic logic: ‘Does the validity of the various epistemic axioms relative to the discovery method depend on enforcing methodological recommendations or structural features of the learning mechanisms?’

However enlightening the classification of epistemologies based upon the first versus third person dichotomy may be, it leaves out something important: the *multi-agent* approach which has recently become a very active area of interdisciplinary research. Multi-agent systems whose members can swap their roles exhibit structures which have been little explored up to now outside game-theory. VFH pays due attention to this newcomer in the field of epistemology. His careful examination of the weaknesses of the classical model of
economic exchange reveals that we have to substitute common belief for common knowledge in order to regain the capacity of explaining how ‘we can agree to disagree’.

VFH should be praised for bringing to the foreground another neglected side of cognition: the time we need to acquire knowledge through learning. To do justice to the diachronic dimension of knowledge, VFH was led to introduce new valid principles of epistemic logic such as (1) and (2) in which ‘F’ means ‘sometimes in the future’ and ‘G’ means ‘always in the future’: (1) $K_{\delta} h \supset FK_{\delta} h$ and (2) $K_{\delta} h \supset GK_{\delta} h$.

It is worth stressing that the hypotheses $h$ considered here should not be construed as Quinean eternal sentences but rather as statements fluctuating in a variety of ways in truth-value over time (Hendricks & Pedersen 99a and 99b), i.e. as fluents, to use a term coined by A.I. people. VFH’s preoccupation with time led him to re-examine the validity conditions of the usual axioms of knowledge and belief.

The transmissibility problem of the $K$-operator raises the following problems: is the axiom $K_a K_b h \supset K_a h$ valid whatever the index we adjoin to the $K$-operator? Some authors have argued that the axiom holds for ‘strong knowledge’ but not for ‘weak knowledge’ (Weingartner 1982, Gillet et Gochet 1991). Without re-opening this discussion here let me point out the following counter-example: from ‘$a$ learned at first hand that $b$ learned at first hand that $h$’ it does not follow that ‘$a$ learned at first hand that $h$’. Should we drop the axiom? Not necessarily. We could blame the failure of the axiom on the occurrence of adverbial expressions such as ‘at first hand’ or ‘directly’ which create hyperintensional contexts.

VFH points out that the above-mentioned axiom can be derived from $T$, necessitation and the $K$-axiom. This axiom is crucial for the transmission of knowledge by teaching. It allows us to draw this inference: Agent $a$, who has read a report, knows that agent $b$, who has performed an experience with litmus paper, knows that the solution is acid. Hence $a$, who has read a report, knows that the solution is acid.

VFH rightly stresses that different propositional attitudes such as ‘accepts as empirically adequate that’ and ‘knows that’ cannot be swapped. From ‘$a$ accepts as empirically adequate that $b$ knows that $\varphi$’ it follows that ‘$a$ accepts that $\varphi$’, but from ‘$a$ knows that $b$ accepts as empirically adequate that $\varphi$’, it does not follow that ‘$a$ knows that $\varphi$’. The reason is that ‘$a$ accepts as empirically adequate’ does not abide by axiom $T$.

Taking advantage of the property of inducement, [the fact that assessment and discovery methods can construct or simulate each other by watching over their respective
outputs on the evidence received], VFH succeeds in mitigating the over-emphasized contrast between assessment and discovery. A detailed formal treatment of this problem can be found in chapter 15 of the author’s former book (Hendricks 2001, 267-278).

In drawing this review to a close I want to stress the breath of coverage and the depth of analysis of Mainstream and Formal epistemology. It is a very original contribution to epistemology, to epistemic logic and to the philosophy of logic. One of the main achievements of the author resides in the systematic application of epistemic logic, computability theory and learning theory to the solution of most of the fundamental problems of mainstream epistemology.

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