Where's the Bridge? Epistemology and Epistemic Logic

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1 Introduction

Epistemic logic begins with the recognition that our everyday talk about knowing and believing has some systematic features that we can track and reflect upon. Epistemic logicians have studied and extended these glints of systematic structure in fascinating and important ways since the early 1960s. However, for one reason or another, mainstream epistemologists have shown little interest. It is striking to contrast the marginal role of epistemic logic in contemporary epistemology with the centrality of modal logic for metaphysicians. This article is intended to help in correcting this oversight by presenting some important developments in epistemic logic and suggesting ways to understand their applicability to traditional epistemological problems. Obviously, by itself, tweaking the formal apparatus of epistemic logic does not solve traditional epistemological problems. Epistemic logic can help us to navigate through problems in a systematic fashion by unpacking the logic of the problematic concepts, it can also lead us to recognize problems that we had not anticipated. This is basically analogous to the role that modal logic has played in contemporary metaphysics.

In the pages that follow three prominent sets of connections between epistemic logic and traditional epistemology will be sketched. Epistemic logic permits formal consideration of the kind of strategies that are available to us in responding to skepticism. It permits a detailed grasp of the social and temporal character of inquiry and of course it allows us insight into the problem of defining the class of scenarios compatible with what someone knows. This last problem is itself equivalent to the problem of explicitly defining the concept of knowledge.

2 The Development of Epistemic Logic

Some epistemic logicians, notably Jaakko Hintikka, are likely to object to our attempt to reconnect epistemic logic to traditional epistemology. For Von Wright, Hintikka and formal investigation has rendered many of the traditional problems and strategies of epistemologists obsolete. Developments in epistemic logic have led some, especially those inclined towards Bayesianism, to claim that 'knowledge' is an overrated notion and that it is unnecessary to the study of action and deliberation. Our view is that the less problematic seeming concept of information that these philosophers prefer, is just as susceptible to the kind of traditional philosophical problems that haunt the epistemologist's notion of knowledge. Three central notions in traditional epistemology are knowledge, belief and doubt; while the three central themes in formal approaches to epistemology are learning, information and strategies. These two sets of concepts are not alien to one another. Instead, as we shall demonstrate, formal treatments of problems related to learning, information and strategies sheds light on parallel problems in the investigation of knowledge, belief and doubt.

Prior to Jaakko Hintikka's seminal *Knowledge and Belief* [Hintikka 62, 05], Rudolf Carnap, Jerzy Łos, Arthur Prior, G.H. von Wright, Nicholas Rescher and others recognized that the our discourse about knowledge and belief exhibits certain systematic regularities that can be presented in an axiomatic-deductive system. Some features of the logical behavior of epistemic concepts are obvious. For instance, claiming to know 'p and q' implies that you know q. However, the lack of an appropriate semantics limited the philosophical usefulness of early reflections on epistemic logic. Advances in intensional and modal logics in the 1950s led Hintikka, von Wright, Lemmon and others to tackle the problem of providing a model theory for such systems. From there, philosophically inclined logicians grappled with the question of how such model theories ought to be interpreted. While epistemic logic inevitably traffics in idealizations of one kind or another, philosophers became increasingly interested, by the 1970s in crafting realistic formal treatments of knowledge. Developments since *Knowledge and Belief*, principally those that have since come since Kutchera's [Kutschera 76] and Lenzen's [Lenzen 78] attempts to integrate modal and epistemic logics have focused on efforts to formally model the dynamical nature of knowledge-systems. Gardenfors' [Gärdenfors 88] account of belief revision was particularly important in setting the stage for a slew of dynamical models of knowledge, some of which we will discuss below. The purpose of reviewing some of these developments is to show how advances at the formal level and traditional epistemological questions intersect in important ways.

Generally speaking, contemporary epistemology is organized around two major goals:

1. The long-standing goal of securing knowledge and simultaneously responding to the challenge of skepticism, and

2. the goal of modelling the dynamics of epistemic and doxastic states.

The first of these goals has, for the most part, been a concern of philosophers who rely on thought experiments, traditional conceptual analysis or intuitions-based methods of various kinds. By contrast, philosophers working with formal tools drawn from logic, probability theory and computer science have pursued the second goal. The apparent divergence of both enterprises can be reconciled to some extent once one recognizes that both goals bear on a third problem, namely

3. the problem of understanding the rationality of inquiry.

This problem, of course, is of equal importance to both mainstream and formal epistemologists. Dynamical treatments of epistemic logic and insights from epistemic logicians into the logic of inquiry speak directly to this third, unifying goal. In recent years, it is precisely the dynamical model of knowledge and inquiry that has concerned philosophically inclined epistemic logicians. The purpose of this article is to draw the attention of less formally inclined epistemologists to fertile advances that have taken place in this area.

However, it is not only recent developments that have bearing on traditional epistemological questions, from its earliest beginnings epistemic logic and epistemology have been inextricably entwined. For instance, while Hintikka's early articulation of epistemic logic in *Knowledge and Belief* is not widely acknowledged for its pertinence to traditional epistemological questions, Hintikka had strong epistemological ambitions even at this early stage. Revisiting *Knowledge and Belief* with the benefit of hindsight reveals the manner in which it systematically recast the three problem areas mentioned above. Developing the logic of knowledge and belief was not, according to Hintikka, merely another technical spin-off of advances in modal and other intensional logics. Its purpose is to ground a logical epistemology by elucidating various epistemic notions and providing a medium for reasoning about them in a systematic manner.

Let's begin with the logic. To obtain the propositional language of knowledge and belief the idea is to syntactically augment the language of propositional logic with two unary epistemic operator K_a and B_a such that

$$K_a p \quad \text{reads} \quad \text{`Agent } a \text{ knows } p\text{'}$$

and

$$B_a p \quad \text{reads} \quad \text{`Agent } a \text{ believes } p\text{'}$$

for some arbitrary proposition p.[1] These formalizations of knowledge and belief are roughly interpretations of $\Box p$ in alethic logic reading 'It is necessary that p'. Interpreting modal logic epistemically and doxastically is crudely a reading of modal formulae as epistemic and doxastic statements expressing attitudes of certain agents towards certain propositions.

[1] The notation for the propositional language used in this presentation diverges slightly from the one in *Knowledge and Belief*.

The semantics of modal logic is likewise given a novel interpretation. Ever since *Knowledge and Belief* Hintikka has offered the following basic intuitive semantic interpretation:

> When you know that S, you can legitimately omit from consideration all possibilities under which it is not the case that S. In other words you can restrict your attention to the situations in which it is true that S. [Hintikka & Halonen 98]

It is natural in our time to think of possibilities in terms of some form of possible worlds semantics. Hintikka has frequently resisted this interpretation of the semantics of epistemic logic, for instance:

> In order to speak of what a certain person a knows and does not know, we have to assume a class ('space') of possibilities. These possibilities will be called scenarios. Philosophers typically call them possible worlds. This usage is a symptom of intellectual megalomania. [Hintikka 03]:19.

Rather than embracing possible worlds talk in its full megalomaniacal glory, the unary operators of knowledge (K) and belief (B) may be read as:

$K_a p \approx$ *in all possible scenarios compatible with what a knows it is the case that p*

$B_a p \approx$ *in all possible scenarios compatible with what a believes it is the case that p*

The basic assumption of Hintikka's approach is that any ascription of propositional attitudes like knowledge and belief, requires partitioning of possible scenarios into two sets depending on the compatibility of the scenario with the attitude in question. In *Knowledge and Belief* the key notion is that of model set rather than possible scenarios or even worlds. Based on the partition, the agent is capable of constructing different 'world-models' using the epistemic modal language. He is not necessarily required to know which one of the world-models constructed is the real scenario. All the same, the agent does not consider all world-models equally possible or accessible given his epistemic state at that instant. Some world-models may be incommensurable with his current information or background assumptions. These incompatible world-models are excluded. In epistemic logic, as in many informal epistemologies, it is typically stipulated that the smaller the set of scenarios an agent considers possible, the smaller his uncertainty.

Thus, epistemic logic offers a way of systematically framing the problem of defining the class of scenarios compatible with what someone knows. However the basic philosophical question concerns the problem of determining what it is that counts as a legitimate partitioning of the set of worlds. As we shall see in the next section, if my only criterion for partitioning is logical consistency then I will find scenarios that are compatible with my model that undermine the very possibility of knowledge. For all I know, I might be in the Matrix or be subject to the machinations of Descartes' evil deceiver. How can I be sure that my inclusion or exclusion of scenarios is legitimate? This last problem, generated by the skeptical challenge, is itself equivalent to the problem of explicitly defining the concept of knowledge.

3 Skepticism and the Use of Force

If you really know something, then new information should not cause you to change your mind. Knowledge is, in this sense, infallible. The classical conception of knowledge as possessing the property of infallibility is taken to require, that for an agent to have knowledge of some proposition he must be able to eliminate *all* the possibilities of error associated with the proposition in question. The set of all worlds is accordingly considered. However, the set of possible worlds is too rich for knowledge to have scope over. This set includes some rather bizarre worlds in which all knowers are systematically in error in one way or another and it might even be taken to include worlds in which contradictions are true. If these worlds were to be considered relevant, skepticism would have the upper hand all the time.[2] Epistemology has long worked to provide a response to skepticism so as to secure the possibility of knowledge. Epistemic logic, as presented by Hintikka, is in much the same business given the centrality of partitioning of scenarios. The partitioning of scenarios into those that can be legitimately ignored and those that are relevant, of course assumes some account of *legitimacy.* Understanding what legitimacy amounts to here is a deep philosophical problem and Hintikka has suggested that it is equivalent to defining knowledge. However, since some of the central properties of any viable account of knowledge, including prominently infallibility, simply cannot be defined with respect to *all* possible worlds, some partitioning of worlds will be required for epistemology to even begin to get started. The strategy of screening off possibilities of error to secure knowledge is a basic tenet of Knowledge and Belief:

> Whoever says "I know that p" proposes to disregard the possibility that further information would lead him to deny that p although he could perhaps imagine (logically possible) experiences which could do just that. [Hintikka 62, 05]: 17.

The 'logically possible' experiences referred to are those pertaining to possibilities of error that any account of knowledge must exclude. These would include conceivable scenarios in which the very possibility of knowledge is undermined: brains in vats, malicious gods and the like. This way of responding to skepticism by limiting the set of citable possible worlds carrying potential error has been dubbed 'forcing' by Hendricks [Hendricks 01] and in particular

[2] For a detailed treatment of the relationship between forcing and skepticism, see [Hendricks & Symons 06].

[Hendricks 06]. When it comes to skeptical arguments that would undermine the very possibility of knowledge, the epistemologist must rely on forcing strategies of various kinds in his or her demonstration that the skeptic's possibilities of error fail to be genuine in the relevant sense. This will be the case no matter what one settles on as a definition of knowledge. In this sense, epistemic logic with its forcing strategy assumes that the skeptic has been defeated and demonstrates the structural manner in which one is obliged to model knowledge.

Epistemic logic explicitly operates with a forcing strategy insofar as it is treated as the partitioning of the space of scenarios compatible with the agent's knowledge determines a certain set over which the epistemic operator is to have scope. Contemporary mainstream epistemologists choose to speak of *relevant* possible worlds as a subset of the set of all possible worlds.[3] The epistemic logician considers an *accessibility* relation between scenarios in a designated class out of the entire universe of possible scenarios. There is no principled difference between relevance and accessibility. Informal epistemologies differ as to how relevance is forced given, say, perceptual equivalence conditions [Goldman 76], counterfactual proximities [Dretske 70], [Nozick 81] or conversational contexts [Lewis 96] circumscribing the possible scenarios. Formal epistemologies differ by the way in which the accessibility relation is defined over scenarios depending of course on the underlying semantics whether based on Kripke-semantics [Kripke 63], neighborhood-semantics [Arló-Costa 02], [Arló-Costa & Pacuit 05] or some other semantic construction [Hendricks 01]. For now, attention is restricted to discrete Kripke-style semantics or Hintikka's model sets.

Forcing and accessibility are intimately connected. To be convinced of this one need only recall that many common axioms governing the epistemic and doxastic operators correspond to certain algebraic properties of the frame in the following sense: A modal axiom is valid in a frame if and only if the accessibility relation satisfies some algebraic condition. For an example, the axiom

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$$K_a p \to p \tag{1}$$

is valid in all frames in which the accessibility relation is *reflexive* in the sense that every possible scenario (or world) is accessible from itself, or $\forall w \in W : Rww$, where w is a possible scenario, W is the set of possible scenarios, and R denotes the binary accessibility relation. (1) is called axiom T,[4] or the *axiom of truth* or *axiom of veridicality*, and says that if p is known by a, then p is true in accordance with the standard tripartite definition of knowledge as true justified belief.

Similarly if the accessibility relation satisfies the condition that

$$\forall w, w', w'' \in W : Rww' \wedge Rw'w'' \to Rww''$$

[3] Explicit forcing proposals in the epistemological literature are sometimes referred to as '*relevant alternatives proposals*'. Cf. Bernecker and Dretske [Bernecker & Dretske 00].

[4] This nomenclature due to Lemmon [Lemmon 77] and later refined by Bull and Segerberg [Bull & Segerberg 84] is helpful while cataloguing the axioms typically considered interesting for epistemic logic.

then the axiom

$$K_a p \to K_a K_a p \tag{2}$$

is valid in all *transitive* frames. (2) is called axiom 4 and is also known as the *axiom of self-awareness*, *positive introspection* or *KK-thesis*. The labels all refer to the idea that an agent has knowledge of his knowledge of p if he has knowledge of p. Other axioms require yet other relational properties to be met in order to be valid in all frames: If the accessibility relation is reflexive, symmetric and transitive, then

$$\neg K_a p \to K_a \neg K_a p \tag{3}$$

is valid. (3) is called axiom 5 also better known as the *axiom of wisdom*. It is the much stronger thesis that an agent has knowledge of his own ignorance: If a does not know p, he knows that he doesn't know p. The axiom is sometimes referred to as the *axiom of negative introspection*.

As opposed to (1)–(3) there is a formula or axiom which is valid in all possible frames

$$K_a(p \to q) \to (K_a p \to K_a q). \tag{4}$$

The axiom amounts to the contentious closure condition for knowledge and is also referred to as axiom K, or the *axiom of deductive cogency*: If the agent a knows $p \to q$, then if a knows p, a also knows q.

These axioms in proper combinations together with the rule of epistemization make up *normal* epistemic modal systems of varying strength depending on the modal formulae valid in the respective systems given the algebraic properties assumed for the accessibility relation. The weakest system of epistemic interest is usually considered to being system **T**. The system includes T and K as valid axioms. Additional modal strength may be obtained by extending **T** with other axioms drawn from the above pool altering the frame semantics to validate the additional axioms. Reflexivity is the characteristic frame property of system **T**, transitivity is the characteristic frame property of system **S4**, equivalence the characteristic frame property of **S5**, etc. From an epistemological point of view, the algebraic properties of the accessibility relation are really forcing conditions.

Modal epistemic axioms and systems may be viewed as measures of infallibility and replies to skepticism. For instance, knowing your own knowledge is a way of blocking the skeptic, but knowledge of your own ignorance in terms of axiom 5 is better still. One contempoary motivation for the plausibility of axiom 5 is in data-base applications: An agent examining his own knowledge base will be led to conclude that whatever is not in the knowledge base he does not know and hence he will know that he does not. The axiom of wisdom or negative introspection is a sort of closed world assumption. A closed world assumption is a forcing assumption if anything is, 'shutting the world down' with the agent, leaving the skeptic nowhere to go. To know the truth, to know of your knowledge, and to know of your own ignorance as in **S5** requires 'full' epistemic access which is exactly why the accessibility relation must be an equivalence relation. A theorem of **S5** is the following

$$\neg p \to K_a \neg K_a p \tag{5}$$

which states that if p is not the case, then a knows that he does not know p—the 'truly Socratic person' as Girle explains [Girle 00], p. 157 knowing exactly how ignorant he is.

A bit more ignorance, a bit more skepticism and accordingly a bit more fallibility is allowed in **S4**. Since axiom **5** is dropped and (5) is no longer a theorem,

$$\{\neg p, \neg K_a \neg K_a p\} \text{ and } \{\neg K_a p, \neg K_a \neg K_a p\}$$

are not inconsistent in **S4**. It is possible for an agent to be ignorant of the fact that he does not know when actually he does know. Put differently, the agent is allowed false beliefs about what is known.

Yet more ignorance and skepticism are allowed in system **T** because while

$$\{K_a p, \neg K_a K_a p\}$$

is inconsistent in **S4**, this set of epistemic statements is not inconsistent in **T**. The agent may thus know something without knowing that he does.

The cognitive rationale of logical epistemology must be something like this: The more properties the accessibility relation is endowed with, the more access the agent has to his epistemic universe, and in consequence the more epistemic strength he will obtain at the cost of stronger forcing assumptions at least for dicrete Kripke or Hintikka semantics with pointwise binary accessibility relations.

4 Revision

Knowledge is not only subject to forcing but equally important is its stability. Continuing the passage from *Knowledge and Belief* cited previously, Hintikka explains:

> If somebody says I know that p in the strong sense, he implicitly denies that any further information would have lead him to alter his view. [Hintikka 62, 05]: 21

So changing one's mind about p implies that one did not know p from the outset. This point about how knowing p implies that one would not change one's mind about p which one may also find in some of Unger's scepticism-friendly early work is eventually what led Kripke to complain that if this were so then knowledge would demand doxastic intransigence—that is, if the agent knows p then he must regard all information which suggests $\neg p$ as being misleading. Kripke never actually committed his thoughts on this matter to print, like much of his thoughts on epistemology in fact, but others have cited them.[5]

Hintikka's approach to the stability of knowledge is a clear example of how formal epistemology can intersect with mainstream. Later, for instance, Stalnaker [Stalnaker 96a] suggested using epistemic logic and belief revision to getting the infamous Gettier-cases [Gettier 63]: Milton knows that p iff Milton

[5] We are indebted to Duncan H. Pritchard for directing attention to this latter point.

believes p and learning no further true information would lead him to change his mind. Given the standard analysis of belief revision, the analysis gives rise to an account of knowledge which validates the modal system **S4.2**.

The idea of combining the logics of knowledge and belief with the dynamics of belief revision theory made for some very significant developments. Alchourrón, Gärdenfors and Makinson's seminal belief revision theory (AGM) from the 1980's is a theory about the rational change of beliefs for expansions, contractions and revisions in light of new (possibly conflicting) evidence [Alchourrón et al. 85], [Gärdenfors 88]. In 1994 de Rijke showed that the AGM-axioms governing expansion and revision may be translated into the object language of dynamic modal logic [de Rijke 94]. Segerberg about the same time demonstrated how the entire theory of belief revision could be formulated in a modal logic.

From the mid-1990s onwards Segerberg merged the static first generation doxastic logic with the dynamics of belief change into 'dynamic doxastic logic' [Segerberg 95]. Doxastic operators in the logic of belief like $B_a p$ may be captured by AGM in the sense that 'p is in a's belief-set T', or $\neg B_a \neg p$ becomes '$\neg p$ is not in a's belief-set T'. Similarly for other combinations of the belief operator with negation. An immediate difference between the two paradigms is that while AGM can express dynamic operations on belief-sets like expansions ('p is in a's belief-set T expanded by D', i.e. $p \in T + D$), revisions ('p is in a's belief-set T revised by D', i.e. $p \in T * D$), and contractions ('p is in a's belief-set T contracted by D', i.e. $p \in T - D$), no such dynamics are immediately expressible in the standard language of doxastic logic. On the other hand, action languages include operators like $[\nu]$ and $\langle \nu \rangle$ which prefixed to a well-formed formula p, $[\nu]p$, respectively $\langle \nu \rangle p$ on Segerberg's interpretation mean that 'after $[every]$ $\langle some \rangle$ way of performing action ν it is the case that p'. By introducing three new operators $[+]$, $[*]$, and $[-]$ into the doxastic language the three dynamic operations on belief-sets may be rendered as $[+D]B_a p$, $[*D]B_a p$ and $[-D]B_a p$.

After revising the original belief revision theory such that changes of beliefs happen in 'hypertheories' or concentric spheres enumerated according to entrenchment Segerberg has provided several axiomatizations of the dynamic doxastic logic together with soundness and completeness results [Segerberg 99a], [Segerberg 99b]. The dynamic doxastic logic paradigm may also be extended to iterated belief revision[6] as carefully studied by Lindström and Rabinowicz [Lindström & Rabinowicz 97] and accommodate various forms of agent introspection [Lindström & Rabinowicz 99].

A related approach drawn up by notably van der Hoek, Linder and Meyer at approximately the same time as the dynamic doxastic logic establishes a new way of distinguishing knowledge from belief [Hoek & Meyer 95]. Actions are responsible for bringing about changes of belief. The distinction between knowledge and belief is not just the static feature of paying homage to axiom

[6] A change in beliefs may either occur once in which case it is a one-shot revision or multiple changes may successively occur in which case it is an iterated revision.

T or not but likewise the dynamic features of defeasibility or indefeasibility. Knowledge is indefeasible under the belief-changing operations of expansion, revision, and contraction, belief is not. van Ditmarsch, van der Hoek and Kooi's new 'dynamic epistemic logic' is partly a continuation of this approach which studies how information changes and how actions with epistemic impact on agents may be modelled [Hoek et al. 03].

5 Rationality and the Idealized Knower

Philosophers have raised the question of whether the logic of knowledge makes any epistemological sense. For instance, in his 1972 article "Is epistemic logic possible?" [Hocutt 72] Hocutt famously challenged the applicability of logic to any realistic account of knowledge. There is no guarantee that a knower will recognize that he is committed to some proposition that is logically equivalent to some proposition to which he or she readily assents. Since this is the case, then, Hocutt suggests, the very idea of an epistemic *logic* is on slippery ground. In the 1970s, the problem of logical omniscience[7] posed similar challenges to the very idea of an epistemic logic. There are at least two ways of responding to these kinds of challenges. One rather unpromising approach is to deny the presupposition that epistemic logic should uphold broader epistemological pertinence. The discipline is not obligated to hook up with more general epistemological concerns ranging from closure conditions to justification, methodology, reliability and rationality, as Lenzen argues:

> The search for the correct analysis of knowledge, while certainly of extreme importance and interest to epistemology, seems not significantly to affect the object of epistemic logic, the question of validity of certain epistemic-logical principles. [Lenzen 78]: 34.

Such an approach would render epistemic logic relatively uninteresting to philosophers and overlooks some of the obvious connections with traditional epistemology discussed above. A preferable response involves maintaining that epistemic logic does carry epistemological significance but in an inevitably idealized sort of way: One restricts attention to a class of rational agents where rationality is defined by certain postulates. Thus, agents have to satisfy at least some minimal conditions to simply qualify as rational. This is by and large what Lemmon originally suggests [Lemmon 59]. One such condition would be that assuming an agent as rational entails that he should know the logical laws. For instance, if the agent knows p and $p \to q$, he should be able to use modus ponens to infer q. Now these 'rationality postulates' for knowledge exhibit a striking similarity with the laws of modal and epistemic logic. One may in turn legitimately attempt to interpret the necessity operator in alethic axioms as a knowledge operator and then justify the modal axioms as axioms of knowledge.[8] While Lemmon constructs the rational epistemic agent directly from the

[7] For an excellent survey of logical omniscience, refer to [Whitsey 03].
[8] For a more detailed discussion of this approach, refer to [Girle 00].

axiomatization of the logic, yet another way of justifying the epistemic axioms is by ways of meaning: Find a plausible epistemological story to tell about the semantics of epistemic logic.

This is the line of thought Hintikka pursued from the outset in *Knowledge and Belief*. Hintikka stipulated that the axioms or principles of epistemic logic are conditions descriptive of a special kind of general (strong) *rationality*. The statements which may be proved false by application of the epistemic axioms are not inconsistent meaning that their truth is logically impossible. They are rather rationally 'indefensible'. Indefensibility is fleshed out as the agent's epistemic laziness, sloppiness or perhaps cognitive incapacity whenever to realize the implications of what he in fact knows. Defensibility then means not falling victim of 'epistemic neglience' as Chisholm calls it [Chisholm 63], [Chisholm 77]. The notion of indefensibility gives away the status of the epistemic axioms and logics. Some epistemic statement for which its negation is indefensible is called 'self-sustaining'. The notion of self-sustenance actually corresponds to the meta-logical concept of validity. Corresponding to a self-sustaining statement is a logically valid statement. But this will again be a statement which is rationally indefensible to deny. So in conclusion, epistemic axioms are descriptions of rationality. This argument is spelled out in detail by Hilpinen [Hilpinen 02].

There is an argument to the effect that Hintikka early on was influenced by the autoepistemology of G.E. Moore [Moore 59] and especially Malcolm [Malcolm 52] and took, at least in part, their autoepistemology to provide a philosophical motivation for epistemic logic. There is a twist to this motivation which is not readily read out of Moore's autoepistemology. Epistemic principles may be interpreted as principles describing a certain strong rationality. The agent does not have to be aware of this rationality, let alone able to immediately compute it from the *first person perspective* as Hintikka argues:

> In order to see this, suppose that a man says to you, 'I know that p but I don't know whether q' and suppose that p can be shown to entail logically q by means of some argument which he would be willing to accept. Then you can point out to him that what he says he does not know is already implicit in what he claims he knows. If your argument is valid, it is irrational for our man to persist in saying that he does not know whether q is the case. [Hintikka 62, 05]:

In Hintikka's logical system knowledge is closed in the sense of (4). The closure is needed for driving the argument through even if the local agent is not immediately computing it. 'I get by with a little help from my friends' applies here. The man in the local street may turn out to be rational after all when properly advised of directions *actually* available to him vindicating the first person interpretation. The distinction between the first and third person perspectives on inquiry, which today is in vogue, is adressed at length in *Knowledge and Belief*.[9]

[9] For more on this distinction and its epistemological impact, refer for instance to [Hendricks 06].

There is another argument for closure that Hintikka could make use of. If knowledge is closed in uniform contexts as Lewisian contextualism has it, then this seems to be exactly what Hintikka could say when presented with the closure challenge and the skeptical invitation. The argument for closure so far rests on autoepistemological and rationality considerations but does not necessarily escape Nozick's classical argument against closure. Hintikka has always emphasized the importance of partitioning the set of scenarios into the two distinct compartments consisting of the worlds in accordance with the attitude and the ones not. The scenarios in accordance with the epistemic attitude may be read in accordance with Lewis' context-sensitive quantifier restriction on knowledge [Lewis 96]. Then, the demon scenario, brain-in-a-vat scenario and other derivatives of global underdetermination are simply excluded from the compatibility partition; these extravagant scenarios are not in accordance with the epistemic attitude. Thus, these error-possibilities will not disturb the context, or in Hintikkian terms, will not pass over into the compatibility partition, so knowledge is closed for a given compatible partition, i.e. uniform context.[10]

Textual evidence for the autoepistemological interpretation is available at some striking points in the argument of *Knowledge and Belief*. For instance, while Hintikka argues for the plausibility of the KK-thesis as a governing axiom of his logic of knowledge he directly appeals to Malcolm's 'strong sense of knowing':

> This is especially interesting in view of the fact that Malcolm himself uses his strong sense of knowing to explain in what sense it might be true that whenever one knows, one knows that one knows. In this respect, too, Malcolm's strong sense behaves like mine. [Hintikka 70]: 154.

Besides the requirement of closure and the validity of the KK-thesis, axiom T is also valid to which neither Moore nor Malcolm would object. A logic of autoepistemology seems philosophically congruent with Hintikka's suggestion for a **S4** epistemic logic describing strong rationality.[11]

6 Active Agency and Inquiry

In addition to the question of how one understands the model theory of epistemic logic, another important difference between epistemic logic and other intensional logics is the addition of the agent a to the syntax. The interesting epistemological question is what roles are assigned the agents. The agents are the ones who

[10] This argument is spelled out in greater detail in [Hendricks 04]. The epistemological plausibility of axiom K together with problem of logical omniscience is discussed at length in [Hendricks 06].

[11] The validity of the KK-thesis is controversial, and Hintikka's suggestion for its validity did not go unchallenged. The principle of positive introspection (together with the principle negative introspection and other axioms of epistemic logic) has occasioned many philosophical debates, too numerous to list, much less discuss, here.

apparently have knowledge which is, say, **S4.3** valid. That agents hold the knowledge is also the natural understanding of the symbolic notation $K_a p$ as Hintikka subsequently has emphasized time and again:

> Epistemic logic begins as a study of the logical behavior of the expression of the form 'b knows that.' One of the main aims of this study is to be able to analyze other constructions in terms of 'knows' by means of 'b knows that.' The basic notation will be expressed in the notation used here by 'K_b.' This symbolization is slightly misleading in that a formula of the form $K_b S$ the term b for the agent (knower) is intended to be outside the scope of K, not inside as our notation might suggest. [Hintikka & Halonen 98]: 2.

In *Knowledge and Belief* there only a single role reserved for the agents. They serve as indices on the accessibility relation between scenarios. Epistemic-logical principles or axioms building up modal systems are relative to an agent who may or may not validate these principles. Indices on accessibility relations will not suffice for epistemological and cognitive pertinence simply because there is nothing particularly epistemic about being indices. The agents are *inactive* in what Hintikka recently dubbed the 'first generation epistemic logic' [Hintikka 03].[12] The first generation epistemic logic is roughly characterized by the ambition that cataloguing the possible complete systems of epistemic and doxastic logics would allow for choosing the most 'appropriate' or 'intuitive' ones(s).

If epistemic logics are not to be pertinent to the knower who are they to be pertinent to? An agent may have knowledge which is **S4.3** valid. What bakes the epistemological noodle however is *how* the agent has to *behave* in order to gain the epistemic strength that he has. We need to activate the agents in order to make epistemic logic pertinent to epistemology, computer science, artificial intelligence and cognitive psychology. The original symbolic notation of a knowing agent also suggests this: An agent should be inside the scope of the knowledge operator—not outside. Inquiring agents are agents who read data, change their minds, interact or have common knowledge, act according to strategies and play games, have memory and act upon it, follow various methodological rules, expand, contract or revise their knowledge bases, etc. all in the pursuit of knowledge. Inquiring agents are *active agents* [Hendricks 02], [Hendricks 04], [Hendricks 06].

This is an interpretation of one of the characterizing features, and great virtues of, what Hintikka calls the 'second generation epistemic logic' [Hintikka 03]: The realization that the agents of epistemic and doxastic logic should play an active role in the acquisition, validation and maintenance processes of knowledge and belief. Hintikka observes this obligation by emphasizing the strategies for his new application of epistemic logic as a logic of questions and answers

[12] Active and inactive agenthood was first discussed in a paper 'Active Agents' and the current discussion follows along these lines [Hendricks 02]. Reference to the agent is sometimes dropped in the formalism of epistemic logic such that $K_a p$ becomes Kp and is read 'It is known that p' exactly due to the inactive nature of first generation agents. See for instance [Hintikka 03].

and the search for the best questions to ask [Hintikka 99], [Hintikka 03]. In this new setting, logical epistemology augmented with an independence-friendly logic constitute the basis for an interrogative theory of inquiry.[13] Answers to questions are in essence requests for knowledge, information or epistemic imperatives. Hintikka's approach rests on the recognition that questions are essentially epistemic, insofar as they express epistemic aims. A question's epistemic aim can be presented as a statement specifying the epistemic state which the answer will bring about; the *desideratum* of a particular question.

Consider, for example, the desideratum of the following question:

1. Is Zoe in the kitchen, the living room, or the garden?

is simply:

2. I know that Zoe is in the kitchen or I know that Zoe is in the living room or I know that Zoe is in the garden.

But of course, this is equivalent to stating that

3. I know whether Zoe is in the kitchen, the dining room or the garden.

Hintikka reduces the study of questions to the study of their desiderata. Desiderata can, of course be studied by using our usual traditional logical methods. Desiderata differ from their corresponding direct questions insofar as they crucially involve the term "know" in such a way as to make any viable logic of questions and answers ineliminably epistemic.

Hintikka understands his interrogative model as a game against nature, or against any source of answers to our inquiries. He distinguishes two different kinds of rules or principles characteristic of a game. The *definitory* rules define the game. In a game of chess, for instance, the definitory rules tell us which moves are permitted and which not, what checkmate, castling, mean, and so on. These rules define the game of chess. If a player makes a move not allowed by the definitory rules, say by moving a pawn three spaces forward, it is not a chess move and the player must take it back. We can thus describe the definitory rules of any game or rule-governed, goal-oriented activity. However, knowing the definitory rules of a game does not mean you know how to play. You must also know what Hintikka calls the *strategic* rules (or principles) of a game. In chess, for instance, you must plan your moves, select the best course of action, make judgments as to which moves will serve you better than others, and so on. These rules are not merely heuristic. They can be formulated as precisely as the

[13] Independence-friendly logic (or IF-logic for short) is a first-order logic augmented with an independence operator '/'. The slash notation for a quantified statement of the form Q_2y/Q_1x expresses the independence of the two quantifiers. This independence may be captured by game-theoretical semantics as informational independence in the sense that the move performed or mandated by Q_2y is independent of the move performed Q_1x. Introducing the independence operator then allows for the unequiviocal formulation of a fan of questions and answers without scope ambiguity, cross-world identity problems etc.

definitory rules. This is well explained by the crucial role of *complete strategies* in von Neumann's game theory.

The results of applying Hintikka's distinction to the interrogative 'games' of inquiry are striking. First, the standard rules of an interrogative game—the rules for logical inference moves as well as interrogative moves—are definitory. They tell us nothing about what to do in a logical or epistemological game. The rules for making both logical inference moves and interrogative moves merely define our game. For example, the so-called rules of inference in deductive logic are neither descriptive nor prescriptive but merely permissive, in so far as they do not tell us which particular inference or set of inferences we should draw from a given number of potential premises. What we need, if our inquiry is going to be successful, is more than the definitory rules of inquiry. We need *strategic rules*. Indeed, the better our strategic rules, the better our inquiry. The best player in a game of inquiry is the player with the best strategy, which corresponds in game theory to what happens where values, i.e., "utilities," are associated not with moves themselves but, rather, with *combinations*} of *strategies*, as in von Neuman's game theoretical notion of a *complete strategy*.

Game theory provides a formal framework for reflecting on the nature of a strategy—and it is an agent who may or may not have a winning strategy among other agents. Van Benthem, Fagin, Halpern, Moses and Vardi, Aumann, Stalnaker and others studying game theory have demonstrated how logical epistemology uncovers important features of *agent rationality*. They also show how game theory adds to the general understanding of notions like knowledge, belief and belief revision.[14] Belief revision theorists like Levi, Rott and others model 'informational economy' or 'conservatism' and consider cognitive economics and the problem of rational choice for *agents* [Levi 91], [Rott 03]. Baltag, Moss, Solecki combine epistemic logic with belief revision theory to study actions and belief updates in games [Baltag et al. 99].[15] Another way to add an active perspective to epistemic logic is pursued in non-monotonic logic starting notably with Reiter's default logic and R.C. Moore's autoepistemic logic and later developments thereof

Active agents require multi-agent setups. Hintikka does not provide a multi-agent framework in *Knowledge and Belief* but does consider the transimissibility of knowledge from one agent to another as an iterated version of axiom T with different agent indicies, i.e. $K_a K_b p \rightarrow K_a p$. Transmissibility may be generalized to a general question of learnability between agents applying the same (or different) methods of inquiry based on either assesment or discovery (or possibly other inference engines). [Hendricks 01].

One way to obtain a multi-agent system is to syntactically augment the language of propositional logic with n knowledge operators, one for each agent involved in the group of agents under consideration [Fagin et al. 95]. The primary difference between the semantics for a mono-agent and a multi-agent semantics is roughly that n accessibility relations are introduced. A modal system for n

[14] van Benthem has also pointed out that there is an epistemic logic hidden in game-theory [van Benthem 00].

[15] The work of Baltag, Solecki and Moss was somewhat preceded by [Plaza 89].

agents is obtained by joining together n modal logics where for simplicity it may be assumed that the agents are homogenous in the sense that they may all be described by the same logical system. An epistemic logic for n agents consists of n copies of a certain modal logic. In such an extended epistemic logic it is possible to express that some agent in the group knows a certain fact, that an agent knows that another agent knows a fact etc. It is possible to develop the logic even further: Not only may an agent know that another agent knows a fact, but they may all know this fact simultaneously. From here it is possible to express that everyone knows that everyone knows that everyone knows, that ... . That is *common knowledge*.

A convention would hardly be looked upon as a convention if it was not for common knowledge among the agents to observe the convention as Lewis has noted. Other norms, social and linguistic practices, agent interactions and games presuppose a concept of common knowledge. A relatively simple way of defining common knowledge is not to partition the group of agents into subsets with different common 'knowledges' but only to define common knowledge for the entire group of agents. Once multiple agents have been added to the syntax, the language is augmented with an additional operator C. Cp is then interpreted as 'It is common knowledge among the agents that p'. Well-formed formulas follow the standard recursive recipe with a few, but obvious, modifications taking into account the multiple agents. An auxiliary operator E is also introduced such that Ep means 'Everyone knows that A'. Ep is defined as the conjunction $K_1p \wedge K_2p \wedge ... \wedge K_np$.

To semantically interpret n knowledge operators, binary accessibility relations R_n are defined over the set of possible scenarios W. A special accessibility relation, R°, is introduced to interpret the operator of common knowledge. The relation must be flexible enough to express the relationship between individual and common knowledge. The idea is to let the accessibility relation for C be the transitive closure of the union of the accessibility relations corresponding to the singular knowledge operators. The model $\mathbb{M}$ for an epistemic system with n agents and common knowledge is accordingly a structure $\mathbb{M} = \langle W, R_1, R_2, ..., R_n, R^\circ, \varphi \rangle$ where W is a non-empty space of possible scenarios, $R_1, R_2, ..., R_n, R^\circ$ are accessibility relations over W for which $R^\circ = (R_1 \cup R_2 \cup ... \cup R_n)^\circ$ and φ again is the denotation function assigning worlds to atomic propositional formula $\varphi : atom \longrightarrow P(W)$. The semantics for the Boolean connectives remain intact. The formula K_ip is true in a world w, i.e. $\mathbb{M}, w \models K_ip$ for agent i, iff $\forall w' \in W$: if R_iww', then $\mathbb{M}, w' \models p$. The formula Cp is true in a world w, i.e. $\mathbb{M}, w \models Cp$, iff $R^\circ ww'$ implies $\mathbb{M}, w' \models p$. Varying the properties of the accessibility relations $R_1, R_2, ..., R_n$ as described above results in different epistemic logics. For instance system **K** with common knowledge is determined by all frames while system **S4** with common knowledge is determined by all reflexive and transitive frames. Similar results are possible to obtain for the remaining epistemic logics.

A widely used and very illustrative dynamic embodiment of multi-agent *systems* can be found in [Fagin et al. 95]. In such a multi-agent system each individual agent is considered to being in some *local state*. This local state holds all the information available to the individual agent 'now'. The whole system as the sum of the local agents is in some *global state*. A system like this is a dynamic entity given the global state of the system and local states of the involved agents for any particular time. The dynamics may be modelled by defining what is referred to as a *run* over the system which really is a function from time to global states. The run may in consequence be construed as an account of the behavior of the system for possible executions. This gives rise to *points* which are pairs of runs and times. For every time, the system is in some global state as a function of the particular time. The system may be thought of as series of runs rather than agents. What is being modelled here are the possible behaviors of the system over a collection of executions.

A system like the one just described defines a Kripke-structure with an equivalence relation over points. The accessibility relation is specified with respect to possible points such that some point w' is accessible from the current point w if the agent is in the same local state at w and w'. Knowledge is defined with respect to the agents' local state. Truth of a formula is given with respect to a point. If truth is relative to a point then there is a question of *when* which opens up for the introduction of *temporal operators*. One may for instance define a universal future-tense operator ('$\Box$' in their notation) such that a formula is true relative to the current point and all later points.[16] The mixture of epistemic and temporal operators can handle claims about the temporal development of knowledge in the system. A multi-modal axiom considered in [Fagin et al. 95] is the following one: $K_a p \to \Box K_a p$.The axiom says that if an agent a knows p at some particular point, then he will know p at all points in the future. The combined axiom holds under special circumstances.[17]

In multi-agent systems like the one just described it is possible to endow the agents with *epistemic capacities* facilitating special epistemic behaviors. Fagin, Halpern, Moses and Vardi consider 'perfect recall': Interacting agents' knowledge in the dynamic system may increase as time goes by but the agents may still store old information. The agent's current local state is an encoding of all events that have happened so far in the run. Perfect recall is in turn a methodological recommendation telling the agent to remember his earlier epistemic states.

There are other interesting structural properties of agents being studied in the literature of dynamic epistemic logics. In an epistemic logic suited for modelling various games of imperfect information van Benthem refers to such properties as 'styles of playing' [van Benthem 00]. Properties like 'bounded memory', various 'mechanisms for information updates' and 'uniform strategies' are analyzed in [van Benthem 01];[18] perfect recall and 'no learning' are studied by van Ditmarsch, van der Hoek and Kooi as they relate to the change of knowledge given the execution of certain plans [Hoek et al. 03]. These and

[16] Other temporal operators may be defined as well, see [Fagin et al. 95].

[17] There are other ways of combining epistemic and tense modalities; for a different approach and an overview refer to [Hendricks 01], [Hendricks 06].

[18] For an excellent survey of the logic in games refer to van Benthem's recent lecture notes [van Benthem 00a].

other properties of the agents are making them active agents.

7 Conclusion

The modelling 'record' for second generation logical epistemology is impressive: Multiple epistemic operators, multiple doxastic operators, common knowledge operators, temporal operators, mono-modal systems, multi-modal systems, dynamic modal systems, epistemic capacities of active agents, and this is not an exhaustive list. There is a vast range of applications and models that use these advanced epistemic logics. Examples range from robots on assembly lines, social and coalitional interactions in 'social software', models of public announcement, card games, 'live' situations in economics, miscellaneous linguistic practices and so on.[19] However, our purpose in this article has not merely been to summarize the achievements of epistemic logic, but to show its relevance to traditional epistemological questions.

The three central notions in mainstream epistemology are knowledge, belief and doubt; and three equally central notions in formal epistemology are learning, information and strategies. It should be relatively clear at this stage that these two sets of notions are congruent and parallel. As we mentioned in the introduction, developments in epistemic logic might have led some to claim that 'knowledge' is an overrated notion and that it is in fact unnecessary for action and deliberation, nevertheless the concept of information is inevitably called upon, and one will obviously prefer information acquired via some reliable method, whether information is obtained by an individual method, a public announcement, or an answer to a question based on some set of useful strategies. In our view, formal treatments of information, inevitably run into parallel kinds of questions and problems that faced traditional epistemology. For example, it might turn out that there is no reliable method available for gathering information, or that no public answer can be trusted, or that there is no strategy available either because of the agent's limited epistemic capacities, or because of restrictions put forth by the epistemic environment. The agent is either left with no, or unreliable acquired information, may lose in a game against nature or against other players. Cases where there is no learning or reliable source of information, no winning strategy, or simply failure in the game of information seeking are equivalent to traditional skeptical dead ends. Likewise, the traditional ideal of knowledge can be thought of as equivalent to successful learning and information retrieval.

By emphasizing the equivalent conceptual features of formal and traditional terms we are not claiming that they are synonymous in any straightforward sense. There are, of course, basic conceptual differences between them. Rather, we are suggesting that the logical features of these notions are at least similar enough that fertile interplay is almost inevitable. For instance, it would be

[19] 'Social software' is a term coined by Rohit Parikh denoting the use of methods from epistemic logic, game theory, belief revision and decision theory to study social phenomena. For more, refer to [Hendricks et al. 05].

strange indeed if the study of learning mechanisms shed no light on knowledge and belief acquisition. As this article has shown, the interplay between epistemic logic and epistemology works dialectically. Just as the formal study sheds light on traditional discussions, informal analyses of the nature of knowledge and belief have shaped formal analyses of the mechanisms responsible for the reliable acquisition.

It can be concluded that that Lenzen's willingness to let epistemic logic and traditional epistemological concerns go there separate ways runs the risk of neglecting the interesting and fertile feedback that is possible between these two enterprises. Obviously there are important methodological differences between the two, but even our limited review of the history of epistemic logic demonstrates that there is enough applicability to keep philosophers busy and interested for a long time. The ongoing dialectic interplay between epistemic logic and epistemology sketched here forms the basis for numerous bridges between traditional and formal epistemology.

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