2 ACTIVE AGENTS

▸ Vincent F. Hendricks

Department of Philosophy
and Science Studies
Roskilde University
vincent@ruc.dk

Epistemic logic can be tracked back to the Middle Ages. The subject was really boosted to the attention of logicians and philosophers by Von Wright's first systematic surveys in the 1950's. Roughly 10 years later Hintikka published his seminal work on the logic of knowledge and belief. Epistemic and doxastic logics have since then grown into powerful enterprises enjoying many important applications.[1] The purpose of this paper is twofold:

1. to place some central themes of epistemic logic in a general epistemological context, and

2. to outline a new framework for epistemic logic developed jointly with S. Andur Pedersen unifying some key 'mainstream' epistemological concerns with the 'formal' epistemological apparatus.

[1] To track the history and development of epistemic logic from the first formulations to its contemporary forms; to consider some of the many applications in philosophy, computer science, game theory, economics, linguistics etc.; and to discuss the developments of epistemic logic in multi-modal systems were the conference aims of the first ΦLOG conference, *Dimensions in Epistemic Logic*, held at Roskilde University, Denmark, May 2-4, 2002. The conference featured lectures by some of the most important contributors to epistemic logic over the years including Joseph Halpern (USA), Jaakko Hintikka (USA), Wiebe van der Hoek (NL), Wolfgang Lenzen (DE), Hans Rott (DE), Krister Segerberg (SE), John Sowa (USA), Moshe Vardi (USA) and Ryszard Wojcicki (PL).

In 1970, when epistemic logic was still partly in its infancy, Scott prophetically noted:

> Here is what I consider one of the biggest mistakes of all in modal logic: concentration on a system with just one modal operator. The only way to have any philosophically significant results in deontic logic or epistemic logic is to combine these operators with: Tense operators (otherwise how can you formulate principles of change?); the logical operators (otherwise how can you compare the relative with the absolute?); the operators like historical or physical necessity (otherwise how can you relate the agent to his environment?); and so on and so on. [45], p. 143.

The criticism is obviously quite severe both theoretically and for applications. In recent years however modal logicians have begun to take Scott's perennial criticism to heart. In branching tense logic researchers are mixing alethic and tense logical operators [4], [49]. In the epistemic logic of Fagin, Halpern, Moses and Vardi, epistemic operators are combined with tense logical operators while modelling the knowledge dynamics of an entire system of agents [6]. Sowa's theory of nested graph models based on Dunn's semantics for laws and facts combined with a theory of contexts is a way of simplifying the reasoning tasks in multi-modal reasoning [46], and can be adapted to Kripke models, situation semantics, temporal models and variants of these.

Epistemic variants of modal logics usually begin with the slippery and much debated notion of 'possible worlds':

> In order to speak of what a certain person a knows and does not know, we have to assume a class ('space') of possibilities. These possibilities will be called scenarios. Philosophers typically call them possible worlds. This usage is a symptom of intellectual megalomania. [25], p. 19.

A difference between a philosophical logician and a philosopher is that while the logician often remains rather agnostic about the ontological significance of the possible worlds and may just refer to them as scenarios, situations, states, contexts or conceptual constructions, the philosopher is usually quite concerned with the metaphysical baggage that comes along with the notion.

Possible worlds are often viewed as largely unanalyzed or even unanalyzable entities complete in their spatio-temporal history whatever they are. Another way to fulfill Scott's wishes is to decompose or 'deconstruct' possible worlds, provide them with some structure that allows one to model and study their temporal extensions, their spatial extensions and the agent to which these extensions are epistemically pertinent. This is in a way what Sowa is doing when levelling contexts and another way of decomposing worlds and providing them with explicit structure is accomplished by *modal operator theory.*

Modal operator theory (a term coined in [15], [18], [19]) is the cocktail obtained by mixing epistemic, tense and alethic logic with some concepts drawn from formal learning theory[2] and has so far been used to study the validity of limiting convergent knowledge.

The initial setup is adopted from Kelly's computational epistemology in [29]. First, an evidence stream is what supplies a scientific inquiry method or agent with evidence.[3] An evidence stream, ε, is an *ω-sequence* of natural numbers, i.e. $\varepsilon \in \omega^{\omega}$. According to the definition it is assumed that the method of inquiry is studying some system with discrete states that may be encoded by natural numbers, so that in the limit the method receives an infinite stream ε of numbers. Hence, an evidence stream $\varepsilon = (a_0, a_1, a_2..., a_n, ...)$ consists of *code* numbers of evidence, i.e. at each state i of inquiry ε_i is the code number of all the evidence acquired at this state. Let the lower case Greek letters $\tau, \theta, \zeta, \mu, \varsigma, \upsilon, ...$ denote evidence streams. It is now possible to provide possible worlds with some explicit analyzable structure leaving the metaphysics behind:

> A possible world is a pair consisting of an evidence stream ε and a state coordinate n, i.e., (ε, n), where

[2] Recently redubbed *computational epistemology* by K. Kelly.

[3] The terms 'method' and 'agent' are used interchangeably.

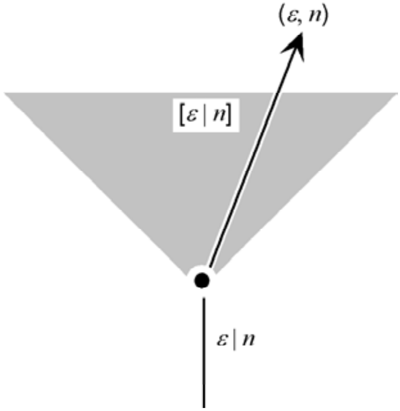

Figure 1 Handle, fan and possible world

$\varepsilon \in \omega^\omega$ and $n \in \omega$.

The set of all possible worlds $\mathcal{W} = \{(\varepsilon, n) \mid \varepsilon \in \omega^\omega, n \in \omega\}$. The following notational conventions are introduced with respect to worlds:

- Let all pairs $(\varepsilon, n), (\tau, n'), (\theta, m), (\varsigma, m''), \ldots$, denote possible worlds.
- Let $\varepsilon \mid n$ denote the finite initial segment of an evidence stream also called the *handle*.
- Let $\omega^{<\omega}$ denote the set of all finite initial segments of evidence streams.
- Let $(\varepsilon \mid n)$ denote the set of all infinite evidence streams that extend $\varepsilon \mid n$, also called the *fan*.

The handle $\varepsilon \mid n$ represents the evidence seen up to state n, i.e. $\varepsilon \mid n = a_0, a_1, \ldots a_{n-1}$. The rest $\varepsilon \backslash \varepsilon \mid n = a_n, a_{n+1}, a_{n+2}, \ldots$, is the evidence that one would observe if the world develops according to (ε, n). The set of possible worlds in the fan is defined by $[\varepsilon \mid n] = (\varepsilon \mid n) \times \omega$. In other words $[\varepsilon \mid n] = \{(\tau, k) \mid k \in \omega \text{ and } \tau \mid n = \varepsilon \mid n\}$. (Figure 1)

The world fan $[\varepsilon \mid n]$ represents the *minimal background knowledge* of possible empirical possibilities or values that the world may take according to the method.

This simple characterization of evidence streams, state coordinates and possible worlds imposes a branching time structure. For any finite time the method has observed some handle of the world, but from that point onwards, the world may 'branch' in any way it pleases for all the method knows. So points or moments in time can be defined as finite initial segments of evidence. This indicates a branching time structure and a branching tense logic rather than a linear model of time. The current structure is closely related to standard Ockhamistic semantics. An Ockhamistic tense structure is typically given by:

1. O is a non-empty set of moments in time,

2. $\prec$ is an irreflexive and transitive earlier-later relation which is backwards linear:

$$\forall n, n', n'' \in O : (n \prec n' \wedge n'' \prec n') \rightarrow (n \prec n'' \vee n'' \prec n \vee n = n'').$$

The notion of *chronicles* plays an important role in Ockhamistic semantics; they may be understood as possible courses of events or evidence and are defined as maximal linear subsets of structure $(O, \prec)$. In general, define the Ockhamistic model to be a triple $(O, \prec, V)$ where V is a valuation function which assigns truth-values $V(n, a)$ to all pairs where the first argument is an element of O and the second argument a is a propositional variable. Then a curious thing arises:

> Now truth is relative to a moment as well as a chronicle to which the moment belongs. The moment with respect to which truth is relative, is the moment where the formula is interpreted. [4], p. 4.

Hence the valuation function V takes three arguments including a specific moment in time n, the chronicle c to which the particular moment belongs and a propositional variable a. It is possible to define an Ochamistic tense structure in modal operator theory as a pair $(\mathbb{T}, \prec)$ such that:

1. $\mathbb{T} = \{\varepsilon \mid n \mid \varepsilon \in \omega^\omega, n \in \omega\}$

2. $t_1 \prec t_2$ iff $\varepsilon \mid n = \tau \mid m$ and $n < m$

where a *moment* in time t is the branching moment of times. In other words, a moment in time is a finite initial segment $t \in (\mathbb{T}, \prec)$. Hence a moment in time is defined as the course of events up until 'now'. Time is based on events and thus the branching time structure is given by events. We are working on the entire tree of finite sequences of natural numbers where finite initial segments (of events) define moments in time.

The evidence stream ε is the actual evidence stream. The state coordinate n, for some specified n, is to be thought of as the 'age' of the world (ε, n), i.e., n is the time in the branch (or the universal clock). The earlier-later relation is defined with respect to finite initial segments.

The structure $(\mathbb{T}, \prec)$ satisfies the requirements above; the relation is irreflexive, transitive and backwards linearly ordered. There exist, of course, many other Ockhamistic tense structures. One example would be a structure that extends into the indefinite past. The current structure, however, has a starting point.

A chronicle c can now be defined as a maximal linear subset of $(\mathbb{T}, \prec)$ with the form

$$c_\varepsilon = \{\varepsilon \mid k \mid k \in \omega\}$$

for some $\varepsilon \in \omega^\omega$. The set of all chronicles is

$$\mathcal{C} = \{c_\varepsilon \mid \varepsilon \in \omega^\omega\}.$$

The agents will eventually need some hypotheses to have knowledge of. Hypotheses will be identified with sets of possible worlds. Define the set of all simple empirical hypotheses

$$\mathcal{H} = P(\omega^\omega) \times P(\omega).$$

So $h \in \mathcal{H}$ iff $h = (a_1, a_2)$ where $a_1 \subseteq \omega^\omega$ and $a_2 \subseteq \omega$. An empirical hypothesis h is said to be *true* in world (ε, n) iff

$$(\varepsilon, n) \in h \text{ and } \forall l \in \omega : (\varepsilon, n + l) \in h.$$

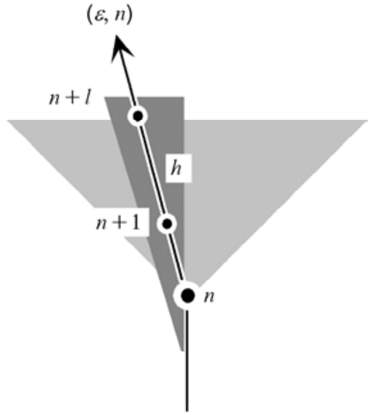

Figure 2 Truth of an empirical hypothesis in a possible world

Truth requires identification and inclusion of the actual world (ε, n) in the hypothesis for all possible future states of inquiry. (Figure 2)

Given the branching time structure, a host of different types of hypotheses are definable with respect to their truth-value fluctuations over time but for now it is assumed that the hypotheses of interest are *absolute time-invariant* empirical hypotheses.[4] In order to define absolute time invariance, define first consistency between evidence and a hypothesis: An empirical hypothesis h is consistent with respect to the evidence iff

$$\exists(\tau, k) : (\tau, k) \in [\varepsilon \mid n] \cap h \text{ and } k \geq n.$$

From here one may define absolute time invariance such that an empirical hypothesis is absolute time invariant with respect to a possible world (ε, n) insofar

$$\text{if } \exists(\mu, k) \in [\varepsilon \mid n] \cap h \text{ then } \exists\tau \in (\varepsilon \mid n) \forall l \in \omega : (\tau, k' + l) \in h,$$

where $k' = \max\{n, k\}$.

Modal operator theory now allows for the introduction of tense and alethic operators in the following intuitive way where the temporal operators F, G, P and H take their usual meaning.

[4] In [6] such hypotheses are called *stable* in truth-value.

1. $(\varepsilon, n) \models Fh$ *iff* $\exists k > n : (\varepsilon, k) \models h$.
2. $(\varepsilon, n) \models Gh$ *iff* $\forall k > n : (\varepsilon, k) \models h$.
3. $(\varepsilon, n) \models Ph$ *iff* $\exists k < n : (\varepsilon, k) \models h$.
4. $(\varepsilon, n) \models Hh$ *iff* $\forall k < n : (\varepsilon, k) \models h$.
5. $(\varepsilon, n) \models \Box h$ *iff* $\forall (\tau, m) \in \mathcal{W} : (\tau, m) \models h$.
6. $(\varepsilon, n) \models \boxdot h$ *iff* $\forall k \in \omega : (\varepsilon, k) \models h$.
7. $(\varepsilon, n) \models \boxminus h$ *iff* $\forall (\tau, m) \in [\varepsilon \mid n] : (\tau, m) \models h$.

The operator '$\Box$' is called universal necessity since it quantifies indifferently over the set of all possible worlds simpliciter. Operator '$\boxdot$' is called temporal necessity because it quantifies indifferently over all times. Finally '$\boxminus$' is called empirical necessity because it ranges over all the empirically possible alternatives relative to the world fan.

Now that both tense and alethic operators have been defined it remains to introduce the epistemic operators in this new framework.

From Indices to Functions

Having defined the space of possibilities, the early epistemic logic – what is referred to as the 'first generation epistemic logic' by Hintikka in [25] – proceeded axiomatically. 'An agent δ knows that p' (where p is some proposition) is formalized as a modal operator $K_\delta p$ in a formal language which is interpreted using the standard apparatus of modal (alethic) logic. One of the hopes was that cataloguing the possible complete systems of such logics would allow for a picking of the most appropriate or intuitive ones often ranging from **S4** over the intermediate systems **S4.2-S4.4** to **S5**.[5] By way of example, in [22] Hintikka

[5] Refer to Gotchet and Gribomont's forthcoming paper for an excellent survey of epistemic logic and its key issues [9].

settled for **S4** while Lenzen in [31] argued for **S4.2**. The significant difference between alethic logic and epistemic logic at the time was the addition of the agent δ to the syntax. The interesting epistemological question is what roles are assigned the agents in the first generation epistemic logic since they are the ones who apparently have knowledge which is, say, **S4.3** valid. That agents hold the knowledge is also the natural understanding of the notation $K_\delta p$:

> Epistemic logic begins as a study of the logical behavior of the expression of the form 'b knows that.' One of the main aims of this study is to be able to analyze other constructions in terms of 'knows' by means of 'b knows that.' The basic notation will be expressed in the notation used here by 'K_b.' This symbolization is slightly misleading in that a formula of the form $K_b S$ the term b for the agent (knower) is intended to be outside the scope of K, not inside as our notation might suggest. [24]

The problem is that the only roles of the agents in the 'first generation epistemic logic' are to serve as indices on the accessibility relation between possible worlds. Now, epistemic-logical principles or axioms building up modal systems are relative to an agent whom may or may not validate these principles or axioms like the KK-thesis. Indices on accessibility relations will not suffice for epistemological and cognitive pertinence simply because there is nothing particularly epistemic about being indices. The agents are *inactive* in the first generation epistemic logic (Figure 3).

This pans out in the more principal discussion of whether there is, or should be, a relation between the formal results of epistemic logic and more general epistemological concerns like justification, methodology, reliability and rationality? Some epistemic logicians were of the opinion that there should be no such relation:

> The search for the correct analysis of knowledge, while certainly of extreme importance and interest to epistemology, seems not significantly to affect the object of epistemic logic, the question of validity of certain epistemic-logical principles. [31]

Figure 3 An inactive agent

In contrast, Hintikka has all along pursued the idea that epistemic logic should aim at hooking up with broader epistemological issues. Early on in [22] Hintikka argued that the axioms or principles of epistemic logic are conditions describing a certain kind of general (strong) *rationality*. Whatever statements can be proved false by application of the epistemic axioms or principles are not inconsistent in the sense that their truth is logically impossible but rather 'indefensible.' Indefensibility is defined by the agent's sloppiness or incapacity in the past, present or future to follow the implications of what he knows. Defensibility is in turn defined as not falling victim of what Chisholm called 'epistemic neglience'. The idea of defensibility provides a hint as to the cognitive status of the epistemic axioms. An epistemic statement for which its negation is simply indefensible is referred to as 'self-sustaining.' Self-sustenance corresponds to the meta-logical concept of validity. In other words, a self-sustaining statement corresponds to a logically valid statement. Thus, a statement which is rationally indefensible to deny. But then the epistemic principles are descriptions of rationality.

If epistemic logics are not to be pertinent to the knower who are they to be pertinent to? An agent may have knowledge which is **S4.3** valid but what one would really like to know – what really bakes the epistemological noodle – is *why* and *how* the agent has to *behave* in order to gain the epistemic strength that it has in terms of validity. We want active agents in order to make epistemic logic pertinent to epistemology, computer science, artificial intelligence and cognitive psychology, and the

Figure 4 An active agent

original formalization of a knowing agent suggests this. Inquiring agents are agents who read data, change their minds, interact or have common knowledge, act according to strategies and play games, have memory and act upon it, follow various methodological rules, expand, contract or revise their knowledge bases, etc. all in the pursuit of knowledge—inquiring agents are *active agents* (Figure 4).

This is admittedly an extended interpretation of one of the characterizing features, and great virtues of, what Hintikka calls the 'second generation epistemic logic' in [25]: The realization that the agents of epistemic logic should play an active role in the knowledge acquisition, validation and maintenance processes. In [25] and elsewhere, Hintikka observes this obligation by emphasizing the strategies for his new game–theoretical semantics, or epistemic logic as the logic of questions and answers and the search for the best questions to ask.

Game-theory is about strategies for winning games—and it is an agent who may or may not have a winning strategy among the other agents. Fagin, Halpern, Moses and Vardi, Auman, Stalnaker and other logicians studying game theory have demonstrated how epistemic logic uncovers important features of *agent rationality* [7]. Additionally in [6] Fagin et al. stipulate (sometimes) for the multi-agent systems that the agents may possess certain *epistemic properties*—in particular *perfect recall*. The idea of perfect recall is that the interacting agents' knowledge in the dynamic system may grow while the agents still keep track of old information. An agent's current local state encodes all that has happened so far in the system run. That is also an aspect of the active agents paradigm in the new trends of epistemic logic. Hoek develops a ‘dynamic epistemic logic’ which studies how information changes, model *actions* with epistemic aspects and provides a new language to reason about these actions. Actions are viewed as multi-agent Kripke models where a precondition function replaces the usual denotation function for propositional variables [26]. Belief revision theorists like Root model ‘informational economy’ or ‘conservatism’ and consider cognitive economics and the problem of rational choice for *agents* [42].

Modal operator theory joins the second generation epistemic logic obligation. The theory has at its base rather than as a derivative the idea that whatever epistemic axioms and epistemic systems are possible to validate for some epistemic operator is *acutely sensitive to the methodological behavior of the agent involved.* This will be clear when it is realized that agents can be viewed as functions rather than indices as in modal operator theory.

Say that a scientific inquiry method, in particular a *discovery method*, conjectures hypotheses in response to the evidence received. More specifically, a discovery method δ is a function from finite initial segments of evidence to hypotheses, i.e.

$$\delta : \omega^{<\omega} \longrightarrow \mathcal{H}.$$

The only immediate criterion of rationality imposed on a discovery method is that it is not allowed to conjecture absurdities. Thus, for any discovery method δ and for any world (τ, n'): $\delta(\tau \mid n') \neq \emptyset$. What remains to be defined is a criterion of successful convergence for a discovery method.

The convergence criterion imposed on discovery methods in [15], [18], [19], [20] is a limiting one. Limiting convergence is not a novel concept. Peirce held the view that it is impossible to say anything about the direction of science in the short run but science may all the same asymptotically approach the truth in the long run [41]. James also argues for limiting convergence because knowledge of general laws is impossible if one is required to say when science has succeeded [28]. Since James and Peirce, limiting convergence has become a more and more respected convergence criterion both in philosophy [3], computer science [10] and in methodology where for an example computational epistemology uses limiting convergence for obtaining certain characterization theorems [29] while Bayesians apply a limiting convergence criterion to facilitate 'almost sure' convergence theorems [14]. Limiting convergence simply means that the method is allowed to vacillate some number of times, which cannot be specified in advance, before it reaches its modulus of convergence. The user of the method, or the agent himself, may not know when this state of stabilization has occurred since no determinate sign of convergence may ever be produced.

Successful limiting discovery in a possible world is now definable together with the discovery method's modulus of convergence. δ discovers h in (ε, n) iff

$$\exists k \forall n' \geq k \forall (\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq h$$

for which the convergence modulus

$$cm(\delta, h, (\varepsilon, n)) = \mu k \forall n' \geq k \forall (\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq h.$$

The method may be subject to various methodological recommendations, programs commands or behavioral patterns. For a well-known example, say that a discovery method δ is *consistent* iff

$$\forall (\tau, n') : [[\tau \mid n'] \cap \delta(\tau \mid n') \neq \emptyset] .$$

Consistency may be strengthened to another recommendation for which the discovery method δ has *perfect memory* iff

$$\textit{if } (\mu, k) \in \delta(\varepsilon \mid n) \textit{ then}$$

$$(\mu \mid n = \varepsilon \mid n) \textit{ and } \forall l \leq k : (\mu, l) \in \delta(\varepsilon \mid n).$$

Furthermore a discovery method δ is said to be *consistently expectant* iff

$$\textit{if } (\mu, k) \in \delta(\varepsilon \mid n) \textit{ then } [k \geq n \textit{ and } (\mu \mid n = \varepsilon \mid n)].$$

Perfect memory and consistent expectation are inconsistent: Suppose $(\mu, k) \in \delta(\varepsilon \mid n)$. If the discovery method has perfect memory, then $\forall l \leq k : (\mu, l) \in \delta(\varepsilon \mid n)$. Now if the method simultaneously is consistently expectant, then $(\mu, k) \in \delta(\varepsilon \mid n)$ implies $k \geq n$, but by perfect memory $(\mu, n-1) \in \delta(\varepsilon \mid n)$. Contradiction! Thus, a discovery method cannot obey perfect memory and consistent expectation at the same time so methodological recommendations may conflict as Nozick emphasizes in [40] and Kelly proves on numerous occasions [29].

It is possible to define a variety of limiting concepts of knowledge in modal operator theory. It suffices for the current purposes to restrict attention to the following concept of knowledge which may be informally portrayed in the following way (for a thorough epistemological motivation of the concept please refer to [15]):

δ may scientifically know *h in the limit iff there exists a possible world which validates δ's knowledge of h. In other words:*

1. *h is true*,
2. *δ conjectures h after some finite evidence sequence has been read and continues to conjecture h in all future.*

Thus, there is a time n, such that for each later time $n' : \delta$ conjectures h at n' in all possible worlds admitted by the background knowledge in which h is true. Now if we modify this definition by telling the method only to conjecture something entailed by the evidence and the background knowledge then we formally end up with:[6]

$(\varepsilon, n) \models K_\delta h$ *iff*

1. $(\varepsilon, n) \in h$ and $\forall l \in \omega : (\varepsilon, n + l) \in h$,
2. $\forall n' \geq n, \forall(\tau, n') \in [\varepsilon \mid n]$:
   (a) $\delta(\tau \mid n') \subseteq h$,
   (b) $(\tau, n') \in \delta(\tau \mid n')$.

This definition of knowledge is very strong, especially given (2b). It implies entailment of the truth by the evidence and the background knowledge. The method is both logical reliable and *infallible*, and hence condition 1 is obsolete in which case one may simply say that $(\varepsilon, n) \models K_\delta h$ iff

$$\forall n' \geq n, \forall(\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq h, (\tau, n') \in \delta(\tau \mid n').$$

One should observe that since the agent is a function from finite initial segments of evidence to hypotheses there is no knowledge without the method conjecturing a hypothesis. Thus, the agent is *actively within the scope of the operator* so $K_\delta h$ in modal operator theory is genuinely faithful to the intended meaning of the standard formalization of the knowledge operator. Note that the Gettier-cases are automatically handled since there is no way the evidence for (h or h') can be undercut if the evidence entails (h or h').

All these set-theoretical intuitions are completely formalizable in a modal propositional logic $\mathcal{L}$ with some modifications to the standard setup. In short, the syntax includes an infinite supply of propositional variables, brackets, the Boolean operators, the unary modal operators $F, G, H, P, \Box, \boxdot, \boxminus, K_\delta$. The construction of well-formed formulas follow the usual recursive recipe. A model $\mathbb{M}$ is a triple $\langle \mathcal{W}, \varphi, \delta \rangle$ where $\mathcal{W}$ is a non-empty set of possible worlds, φ is a denotation function from propositional variables into $P(\mathcal{W})$ and δ a discovery function:

[6] This methodological recommendation is called *infallibility* in [15].

$\delta : \omega^{<\omega} \longrightarrow P(\mathcal{W})$. A few more modifications to the standard setup are required but the reader is referred to [15] for the details.[7]

Once the syntax and semantics are fixed one may then ask the following pair of questions:

1. *Now which epistemic axioms can be validated by an epistemic operator based on the definition of limiting convergent knowledge for discovery methods?*

2. *Does the validity of the various epistemic axioms relative to the method depend upon enforcing methodological recommendations?*

The first question is close to the first generation issue of epistemic logic, the second question a variety of the second generation issue. It is possible to prove the following theorem:

If knowledge is defined as limiting convergence, then knowledge validates **S4** *iff the discovery method has consistent expectations.* [15], p. 203

Neither axioms (T) or (K)[8] require methodological recommendations for their validity; they are valid given the definition of knowledge as it stands. But axiom (4), or the KK-thesis,[9] does require you to entertain a methodological recommendation even if the discovery method is infallible![10] Some more detail pertaining to KK-thesis and the stated theorem will be provided in the last section.[11] It can additionally be shown for instance that perfect memory is an impediment to validating axiom (4). Thus, methodological recommendations may be classified as to

[7] Some formal sloppiness in this presentation will occur for ease of readability. By way of example, in the modal formalization, A denotes the proposition defining a hypothesis h. However not to introduce too much new notation we continue to write h except where explicitly stated otherwise.

[8] Axiom (T): $K_\delta h \rightarrow h$. Axiom (K): $K_\delta(h \rightarrow h') \rightarrow K_\delta h \rightarrow K_\delta h'$.

[9] Axiom (4): $K_\delta h \rightarrow K_\delta K_\delta h$.

[10] Consistency and perfect memory don't halp either in validating **S4**.

[11] By the way, any method defined for limiting convergence is unable to validate **S5**. Intuitively the reason is that the axiom of wisdom, axiom (5): $\neg K_\delta h \rightarrow K_\delta \neg K_\delta h$, requires the method to turn non-convergence into convergence but when knowledge is defined by convergence this is impossible.

whether they are *boosting* in the sense that a methodological recommendation is *conducive* to validating epistemic axioms and systems, *debilitative* in the sense that the methodological recommendation is an *impediment* to validating epistemic axioms and systems, or *neutral* if it is neither boosting or debilitative.[12]

The general lesson to be learned from the theorem is that whatever epistemic axioms or systems the method is capable of validating is very sensitive to the very behavior of the method and the methodological recommendations one may be required to impose on it in order to obtain the desired epistemic strength—active agents again.

Modal operator theory provides a rich framework for other investigations of which a few highlights will be mentioned. Some axioms including tense modalities turn out to be valid like

$$K_\delta h \rightarrow \begin{cases} 1. & FK_\delta h \\ 2. & GK_\delta h \end{cases}$$

which by the way is independently discussed by Fagin, Halpern, Moses and Vardi in [6]. Axioms including tense and alethic modalities may also be investigated for validity—see further [15], chap. 13.

So far attention has been restricted to discovery methods. One may equally well define an assessment α as a function from finite initial segments of evidence and hypotheses to $\{0, 1\}$ where 0 denotes false and 1 denotes truth:

$$\alpha : \omega^{<\omega} \times \mathcal{H} \longrightarrow \{0, 1\}$$

which is a standard method of justification assessing hypotheses in the light of incoming evidence. Successful limiting convergence can be defined for assessment such that α decides h in the limit in (ε, n) iff

1. if h is true, then

$$\exists k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \tau \mid n') = 1,$$

[12] These notions are similar to Kelly's distinction between permissive and restrictive architectures.

2. if h is false, then

$$\exists k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \tau \mid n') = 0$$

with the following limiting convergence modulus:

$$cm(\alpha, h, (\varepsilon, n)) =$$
$$\mu k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \varepsilon \mid n) = \alpha(h, \tau \mid n').$$

In sum, the assessment α verifies h in the limit when h is true and refutes h in the limit when h is false.

It turns out that discovery methods can *induce* assessment methods in the following way:

If a discovery method δ discovers h in (ε, n) in the limit, then there exists a limiting assessment method α which verifies h in (ε, n) in the limit.

This is not hard to see. Assume that δ discovers h in (ε, n) in the limit and let $cm(\delta, h, (\varepsilon, n))$ be its convergence modulus. Define α in the following way:

$$\alpha(h, \varepsilon \mid n) = 1 \textit{ iff } \delta(\varepsilon \mid n) \subseteq h.$$

It is clear that if $n' \geq cm(\delta, h, (\varepsilon, n))$ then for all $(\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq h$. Consequently $\alpha(h, \tau \mid n') = 1$ and therefore

$$cm(\alpha, h, (\varepsilon, n)) = cm(\delta, h, (\varepsilon, n)).$$

Assessment methods can also induce discovery methods along similar lines. This is interesting because a limiting notion of knowledge defined using an assessment method rather than a discovery method also validates **S4**. This information may in turn be used when *knowledge transmissibility* is studied in what has been labelled *multiple method systems* [15]. Knowledge transmissibility was first studied by Hintikka in [22]. Hintikka investigated whether

$$K_a K_b p \rightarrow K_a p \tag{2.1}$$

held for his definition of knowledge where p is some arbitrary proposition and a, b are agents. In a certain sense knowledge transmissibility is rather trivial here because it is essentially the iterated version of axiom (T) with different agents. As long as the agents index the same possible worlds knowledge transmissibility holds in the sense of (2.1). But in the current scheme of things knowledge transmissibility is far from trivial because there are agents or methods of different natures based on either discovery or assessment. Thus we have to consider in all generality whether

$$K_\Theta K_\Xi A \rightarrow K_\Theta h \tag{2.2}$$

is valid for arbitrary inquiry methods $\Theta, \Xi \in \{\alpha, \beta, \gamma, \delta\}$ where γ, δ are discovery methods while and α, β are assessment methods. This means a classification of the transmissibility instances paraphrased as:

Uniform Transmissibility: Is it possible that a discovery method δ having knowledge of the fact that another discovery method γ has knowledge of some hypothesis h, may obtain knowledge of this hypothesis h and similarly for assessment?

Uniform Transmissibility. Is it possible that a discovery method δ having knowledge of the fact that another assessment method α has knowledge of some hypothesis h, may obtain knowledge of this hypothesis h and similarly starting with an assessment method?

It turns out, given inducement, that the answers to both questions are affirmative, a result which partially breaks the back of the classical, and much celebrated, dichotomy between assessment and discovery in the philosophy of science.

Finally, only absolute time invariant or stable hypotheses have been considered so far. A hypothesis h is a *stable* hypothesis iff

1. $h \subseteq \mathcal{W}$,
2. if $(\varepsilon, n) \in h$ then $\forall m \in \omega : (\varepsilon, m) \in h$.

Such a hypothesis may be expressed model-theoretically in the following way: Proposition A defines a stable hypothesis in $\mathbb{M}$ iff

$$\mathbb{M} \models A \rightarrow \boxdot A$$

since a hypothesis is temporally necessary iff $\forall k \in \omega : (\varepsilon, k) \vDash h$ which is equivalent to the definition of a stable hypothesis.

In [20] we consider the knowledge acquisition possibilities of hypotheses which are far from stable due to philosophical problems like meaning variance and relativism. The framework allows for the definitions of various hypotheses based on their truth-value fluctuations over time. By way of example, h is an *eventually stable* hypothesis iff

1. $h \subseteq \mathcal{W}$,
2. if $(\varepsilon, n) \in h$ then $\exists k \forall m \geq k \in \omega : (\varepsilon, m) \in h$.

An eventually stable hypothesis is depicted in figure 5. Given the tense-logical operators proposition A defines an eventually stable hypothesis in $\mathbb{M}$ iff

$$\mathbb{M} \vDash A \rightarrow \neg \boxdot \neg GA.$$

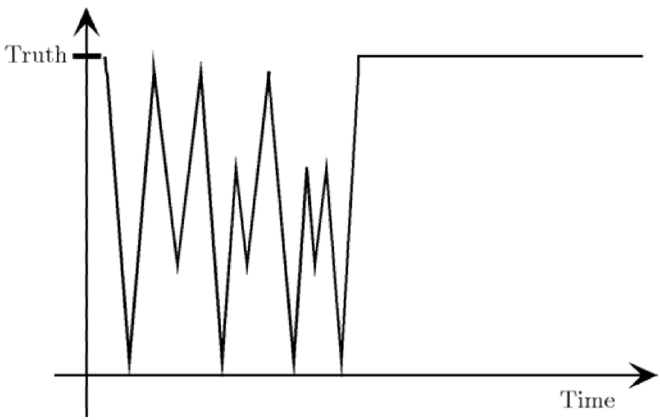

Figure 5 The eventually stabilizing truth-value

Further, say that h is an *oscillating* hypothesis iff

$$\text{if } (\varepsilon, k) \in h \text{ then } \forall n \exists n_1 \exists n_2 : \\ n_1 \geq n \wedge n_2 \geq n \wedge (\varepsilon, n_1) \notin h \wedge (\varepsilon, n_2) \in h.$$

Then proposition A defines an oscillating hypothesis in $\mathbb{M}$ iff

$$\mathbb{M} \vDash A \rightarrow \boxdot(FA \wedge F\neg A).$$

Yet other hypotheses may be defined. This is still work in progress but so far it has been established that some methodological recommendations which otherwise are boosting for the methods in the stable paradigm are debilitative in the relativistic paradigm!

Epistemic Logic and Epistemology

Contemporary epistemological studies are roughly either carried out in

1. a *mainstream* and traditionally rather informal way using largely common-sense considerations and concentrating on sometimes folksy and sometimes exorbitantly speculative examples / counter-examples, or

2. a *formal* way by applying a variety of tools and methods from logic, computability theory or probability theory to the theory of knowledge.

The two traditions have unfortunately proceeded largely in isolation from one another. It may sometimes be hard to see how the formal results hook-up with more traditional epistemological themes. But the two approaches have much in common both epistemologically and methodologically and can significantly benefit from one another.

Forcing

Epistemology is a reply to skepticism. Skeptics have since the days of old cited *prima facie* possibilities of error as the most substantial arguments against knowledge claims. A contemporary response to skepticism is to invoke *forcing*—a term coined in [15]. Forcing is a kind of 'logical sufficiency.' Perhaps it is more of an heuristic principle than really a thesis:

Whenever knowledge claims are challenged by alleged possibilities of error, the strategy is to show that the possibilities of error fail to be genuine in the relevant sense.

By way of examples, the influential *epistemic reliabilism*, Nozick's elegant formulation of the *counterfactual reliabilism* and Lewis' new *modal epistemology* are informal epistemological proposals observing the forcing relation.

Epistemic reliabilism [11], [12], [13] acknowledges the agent's limited cognitive abilities and accordingly deflates the agent's epistemic responsibilities. The idea is to substitute the excessive requirements often proposed by skepticism for knowledge acquisition with more moderate conditions. Hence a justified belief may itself very well be false, but its mode of acquisition must in general yield truth. Besides the requirement that the target belief must be true in order to gain knowledge, its mode of acquisition must rule out all relevant possibilities of error. Thus, the forcing is given by the mode of acquisition: The mode of acquisition may not be able to exclude the possibility that a Cartesian demon is feeding systematically misleading sensations, or that they are vat-generated impressions. Then again these are not realistic possibilities of error.[13] To combat the skeptic it suffices to note that infallible or certainty-producing methods are not required for knowledge.

Nozick's counterfactual reliabilistic knowledge definition [39] (a similar one was advocated earlier by Dretske in [5]) is also a forcing proposal. The inherent decision procedure and the counterfactual semantics require the agent to succeed in all possible worlds sufficiently close to the actual world in order to acquire knowledge. The agent may not know that he is not a brain in a vat—but then again, that possibility of error is so remote. Asking a physicist whether it is viable possibility of error that his voltmeter is calibrated incorrectly while measuring the voltage drop over some LRC circuit is reasonable, asking him then whether it is a realistic possibility of error that he is a brain in a vat is just silly and far out.

[13]... even though the method may not be able to determine this which is the entire point of the brains and the demons. Kelly has shown that global underdetermination exactly like demons and brains equals the impossibility of reliable inquiry [29].

Lewis' modal epistemology assumes knowledge of a great many things [34]. Considering brains in vats and Cartesian demons is to 'epistemologize' which may make the knowledge we thought we had evaporate into thin air. Hence, all we need are rules to slice off possible worlds and then describe how we avoid error and gain truth in the ones left (as humans actually do).

It turns out that a host of formal epistemological proposals share the forcing heuristic as well. Knowledge claims may be restricted by the constraints imposed on the accessibility relation between possible worlds, which may be viewed as the forcing foundation for epistemic logic. Also, probability theory, in particular Bayesian epistemology, requires the agent to succeed in circumstances with high probability and dispose of sets of possibilities carrying low probability [27], [14]. Thus, if the truth lies in worlds ascribed 0 or infinitesimal priors the truth will never be found. That is forcing. Computational epistemology requires success in all worlds in accordance with the background knowledge of empirical possibilities which likewise is to pay homage to the forcing thesis. Finally, modal operator theory requires success in only possible worlds consistent with what has been observed until now which satisfies the forcing relation.

From the forcing perspective what separates the different epistemological proposals, whether mainstream or formal, are the criteria imposed for circumscribing the set of relevant worlds over which to succeed for knowledge to come about. Additionally, if one also observes the difference between what may be called a *first person perspective on inquiry* following Lewis [34] in which

- it is considered what an agent can solve, can do or defend considering the available means for an end given the epistemic environment he is sunk into

and a *third person perspective on inquiry* in which

- it is considered what an agent could solve, could do or defend considering the best means for an end independently of the epistemic environment he is sunk into

the mainstream and formal approaches have even more to say to each other as will become apparent below. A detailed treatment of forcing epistemology is to be found in [16].

Justification and Methodology

Since Plato's *Meno* and *Theatetus* epistemology has sought to identify the necessary and sufficient conditions for the possession of knowledge. The result has been the much celebrated tripartite definition of knowledge as justified true belief.

The justification condition of this definition has probably received the greatest attention of the three components from Plato to contemporary epistemology because it is apparently difficult to define it properly:

> Though there is basic agreement that *something* must be added to true belief to obtain knowledge, what precisely this 'something' is, remains far from being evident. Because of the vagueness of such notions as 'having sufficient reasons for believing', 'being justified in believing', it is difficult to make a decision concerning the adequacy of (5), *i.e.* that knowledge implies justification. [31], p. 28.

In the standard definition, any claim to knowledge requires that the satisfaction of the belief condition is 'adequately' connected to the satisfaction of the truth condition. The two conditions alone are thought to be jointly insufficient to secure knowledge since some true beliefs may be the fortunate result of lucky conjectures, various accidental inferences, evidence collected under obscured perceptual circumstances etc. Now, such beliefs should on the standard analysis obviously not count as knowledge since the belief and truth conditions are inadequately connected to each other due to the questionable *means* by which the *de facto* true beliefs have been derived. According to the justification condition, if some argument or other justificational structure can be provided which describe why the first two conditions are properly connected, then the agent may be said to have secure indication that a known proposition is true.

On top of Lenzen's point of vagueness pertaining to the justificational issue, Gettier's *succés de scandale* counterexamples to the thesis that knowledge is justified true belief did nothing much but to make matters even worse. To avoid the Gettier-cases some mainstream epistemologists have appealed to *reliability*. For an example, Goldman argues that the belief in some hypothesis is justified if and only if the method by which the belief is formed is reliable in the sense of producing more true convictions than false ones in the actual world.[14] Nozick insists on a 'heavier' strategy by appealing to a strongly reliable recursive procedure with a halting condition imposed in all nearby worlds, while Lewis in his modal epistemology speaks of the rule of reliability and refers in a footnote to a type of nomic sufficiency account à la Armstrong. Observe that independently of what reliability is supposed to mean, *it is a criterion imposed on the inquiry method or agent* generating the belief:

The justification condition of epistemology ends up in methodology as the study of how science arrives at its posited truths, i.e. how beliefs are justified by the canons, norms or recommendations and internal workings of the method applied or agent in question.

Whatever is vague about justification has obviously to be resolved and can be resolved methodologically as Sankey recently noted:

> These are questions about the truth-conduciveness of the method. While they relate directly to the epistemic status of the method, they bear *indirectly on the nature of rational justification*. For if use of method conduces to truth, then, given the relation between method and justification, the warrant provided by the method is warrant with respect to truth. [43], p. 1.

Also a mainstream epistemologist like Bonjour stresses a similar point:

> An adequate epistemological theory must establish a connection between its account of justification and its account of truth: *i.e.* it must be shown that justification, as viewed by that theory, is truth-conducive, that

[14] Sometimes Goldman does advocate a many world view but is to this day indecisive on the issue. The (possibly limiting) ratio of true beliefs over false beliefs remains at the core of his reliabilistic theory unfortunately reinviting the Gettier paradoxes which reliability was introduced to block.

> one who seeks justified beliefs is at least likely to find true ones. [2], p. 75.

Methodological recommendations, truth-conduciveness, reliability, convergence, strategies for winning games, changing your beliefs economically and reliably[15] and the like are at the very *core* of many formal epistemological proposals. Computational epistemology scrutinizes the feasibility of recommendations for getting to the truth reliably for both ideal and computationally bounded agents; game-theory models rationality among agents possibly through the use of epistemic logic; belief revision concentrates on informational economy and the agent's rational change of beliefs etc. In general what the mainstream epistemologists are looking for seems to be what the formal epistemologists have to offer.

This article is concluded with an example of the demand and supply situation hopefully revealing the fruitful epistemological and methodological interactions between mainstream and formal theories of knowledge.

KK'ing Diachronically

Also before, but especially after Hintikka's publication of *Knowledge and Belief* in 1962, and to this day, epistemologists have been concerned with two philosophical themes as they relate to *Knowledge and Belief*: (1) the problem of logical omniscience, and (2) the plausibility of the KK thesis both of which are consequences of the logical epistemology advocated in the book.

The logical omniscience problem will not be dealt with here but an elegant treatment of the problem together with a new logic of knowledge and belief for agents with limited reasoning powers and a way of modelling first vs. third person perspectives on inquiry is to be found in Segerberg's recent work [44].

[15] See [30] for a logical reliabilistic analysis of belief revision. Also Martin and Osherson discuss belief revision in the light of computational epistemology [36].

Wojcicki's new approach may also be extended to cover similar issues [48].

The KK plausibility is still unsettled. A mainstream epistemologist like Nozick abandons it because the agent may not be tracking the fact that he is tracking. Not having self-awareness also supports James' distinction between absolutist's philosophy and pragmatism. One may not infallibly know when one has converged to the fact that one has converged to the correct answer. Formal learning theorists like Martin and Osherson are of the same opinion:

> This does not entail that Ψ knows he knows the answer, since (as observed above) Ψ may lack any reason to believe that his hypotheses have begun to converge. [36], p. 13.

In the previous section a concept of knowledge was introduced based on the idea of limiting convergence, and yet limiting convergence is often cited as one of the primary reasons for not validating the KK-thesis! But if one wants to validate the KK-thesis and simultaneously entertain a limiting concept of knowledge how is it possible to have the cake and eat it too?

Given that both tense and alethic modalities can be treated in modal operator theory together with epistemic modalities it makes sense to distinguish between two interpretations of epistemic axioms:

Synchronic axiom: *An epistemic (or doxastic or combined) axiom is* **synchronic** *if the consequent obtains by the very same time the antecedent obtains.*

Diachronic axiom: *An epistemic (or doxastic or combined) axiom is* **diachronic** *if the consequent either obtains later or would have obtained later than the antecedent even if things had been otherwise.*

Malcolm's autoepistemology defends the KK-thesis synchronically from a first person perspective [35]. There are other models of first person knowledge operators validating KK synchronically in particular R.C. Moore [38], Fitting [8] and Arló-Costa [1]. Actually these models yield a stronger logic than **S4**—they validate **S5**. On the other hand, Nozick denies KK at least synchronically from the 1st person point of view. Then there is Levi's epistemological program which essentially is a garden-variety of a first person view in which the main issue in the semantics for Levi is Ramsey's distinction between *the logic of truth* and *the logic of consistency* rather than first and third person distinctions [33]. These two sets of distinctions are obviously related but not exactly identical. In the paper Levi argues against the validity of the KK-principle as an axiom of an epistemic logic of truth which, somewhat simplified, is tantamount to denying that KK is an axiom for a third person knowledge operator. What Levi really argues is that the KK-principle is valid as a principle regulating the consistency of a rational epistemic agent while the logic of truth for epistemic agents need not be regulated by such a principle. Lewis seems to follow suit and their underlying suggestion must in the end be that if there is a universal third person logic of knowledge, such a logic is probably rather weak [34].

With these two interpretations of the epistemic axioms in hand return to the question of how to have the cake and eat it too and the current line of defense for KK.[16] To have knowledge of a hypothesis is to have reached a modulus of convergence after which the method continues to project the conjecture over all later times and possible worlds. Now, knowledge of a hypothesis h is a subset of the hypothesis h. To have knowledge of knowledge of a hypothesis h must be to reach a modulus of convergence only *after* convergence to knowledge of h has arisen. This is because knowledge of knowledge of a hypothesis is a subset of knowledge of a hypothesis, so knowledge of knowledge can only happen once knowledge of the hypothesis has obtained. Hence

$$K_{\Xi}K_{\Xi}h \subseteq K_{\Xi}h \subseteq h. \tag{2.3}$$

One has to *force strategically* to validate KK.

Suppose the method has perfect memory and hence remembers the past evidence. Then KK becomes impossible to validate. Perfect memory demands that the method starts to force for KK at a time l in worlds prior to the modulus of convergence

[16]This is obviously tentamount to the proof of validating axiom (4) of the previous section.

has been reached for mere knowledge of h which only happens at a later stage $n > l$. Forcing for KK prior to knowledge of h is impossible because there may exist a world λ required for KK veering off the actual world ε before the knowledge of h has arisen. Since the method has perfect memory it will attempt to 'crawl' below n to get this world λ in the KK-conjecture. If the method crawls below n and captures λ, then λ will be in $K_{\Xi}K_{\Xi}h$ but not necessarily in $K_{\Xi}h$ and thus violating (2.3) above.

Suppose on the other hand, that the method entertains *consistent expectation*. Consistent expectation implies that any additional convergence and forcing in terms of knowing that one knows h takes place at a point in time $n' > n > l$ where the method *has* converged to h and hence forces already. Adding knowledge to knowledge requires forcing and then later some more forcing. For more detail refer to [15] and [18]

This is fairly easy to see if one stands outside looking in, i.e. if one adopts a third person perspective on inquiry as modal operator theory does. It is less obvious how one could stand in the epistemic environment and know that if one has consistent expectations, then one is eventually going to know that one knows when one does. The validity of KK is up to the method rather than to the world, and hinges on the diachronic interpretation of the KK thesis, but given consistent expectation, *not* unwarranted confidence in the status of one's own earlier beliefs—*the method already knows* if you look at it from the third person point of view, but not necessarily from the first.

The point of introducing the two interpretations of epistemic axioms; the point of distinguishing between first and third persons perspectives on inquiry and the point of forcing are to square away confusion and misunderstandings between mainstream and formal epistemologies. To open up for profitable interactions and exchanges between the two approaches. For example, in criticizing some position, whether formal or informal, without noticing that the criticism is based on a third person perspective and the position advocated is first person may turn out to be criticizing an apple for not being an orange. Similarly if an epistemic axiom is advocated diachronically, then it is not advocated synchronically or indifferently. A mainstream epistemological paradigm may be a forcing strategy, and a formal one may be too—the question is then how they do force respectively putting them on par for comparison.

Admittedly, epistemology becomes significantly more complex with these additional parameters added in and we are going need all the help we can get from formal and mainstream epistemology alike. That's just the way it goes with active agents.

Bibliography

[1] Arló-Costa, H. (1998). 'Qualitative and Probabilistic Models of Full Belief,' *Proceedings of Logic Colloquim'98, Lecture Notes on Logic* **13**, Buss, S., Hajék, P. and Pudlák, R. (eds.), ASL.

[2] Bonjour, L. (1976). 'The Coherence Theory of Empirical Knowledge,' *Philosophical Studies* **30**: 281-312. Reprinted in *Contemporary Readings in Epistemology*, Goodman, M. F. and Snyder, R. A. (eds.). Prentice Hall. New Jersey: Englewood Cliffs (1993): 70-89.

[3] Boyd, R. (1984). 'The Current State of Scientific Realism,' in *Scientific Realism*, Leplin, J. (ed.). Berkeley: University of California Press: 41-82.

[4] Braüner, T., Hasle, P., Øhrstrøm, P. (1998). 'Determinism and the Origins of Temporal Logic,' *Proceedings of the Second International Conference on Temporal Logic*, Applied Logic Series. Dordrecht: Kluwer Academic Publishers.

[5] Dretske, F. (1970). 'Epistemic Operators,' *The Journal of Philosophy* **67**(1970): 1007-1022.

[6] Fagin, R., Halpern, J. Y., Moses Y. and Vardi, M. Y. (1995). *Reasoning about Knowledge.* Cambridge: MIT. Press.

[7] Fagin, R., Halpern, J.Y., Moses, Y. and Vardi, M.Y. (1996). 'Common Knowledge Revisited,' *Annals of Pure and Applied Logic* **96**(1999): 89–105. Reprinted in [21].

[8] Fitting, M. (1983). 'Moore's Non-Monotonic Logic and **S5**,' CUNY.

[9] Gotchet, P. and Gribomont, P. (2003). 'Epistemic Logic,' forthcoming in *Handbook of the History and Philosophy of Logic*, edited by Gabbay, D.M. and Woods, J. Elsevier Science: Amsterdam.

[10] Gold, E. M. (1965). 'Limiting Recursion,' *Journal of Symbolic Logic* **30**: 27-48.

[11] Goldman, A. (1979). 'What is Justified True Belief,' *Justification and Knowledge*, Pappas, G. S. (ed.). Dordrecht: D. Reidel Publishing Company: 1-23.

[12] Goldman, A. (1986). *Epistemology and Cognition.* Cambridge: Harvard University Press.

[13] Goldman, A. (1992). 'Epistemic Folkways and Scientific Epistemology,' in *Liaisons: Philosophy Meets Cognitive and Social Sciences.* Cambridge: MIT Press: 155-175.

[14] Earman, J. (1992). *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory*. Cambridge: MIT Press.

[15] Hendricks, V. F. (2001). *The Convergenc of Scientific Knowledge—A View from the Limit*. Studia Logica Library Series: Trends in Logic, vol. 9. Kluwer Academic Publishers.

[16] Hendricks, V. F. (2003). *FORCING Epistemology*, forthcoming 2003. Berlin: Logos Verlag.

[17] Hendricks, V. F. and Pedersen, S. A. (1998). 'Assessment and Discovery in the Limit Scientific Inquiry,' forthcoming in *Philosophical Dimensions of the Unity of Science—Proceedings of the XI International Congress on Logic, Methodology and the Philosophy of Science*, Cachro, J., Hanuszewicz, S., Kurczewski, G., and Rojszczak, A. (eds.), Dordrecht: Kluwer Academic Publishers, 2002.

[18] Hendricks, V. F. and Pedersen, S. A. (2000). 'KK-ing Diachronically,' presented at *LC2000—European Congress of the Association of Symbolic Logic*, Paris, France, July 23–31, 2000.

[19] Hendricks, V. F. and Pedersen, S. A. (2000). *The Companion to Modal Operator Theory—A Program in Philosophy Online*. Department of Philosophy and Science Studies, Roskilde University, Denmark: `http://www.mot.ruc.dk`.

[20] Hendricks, V. F. and Pedersen, S. A. (2004). *Operators in Philosophy of Science*. In preparation for publication.

[21] Hendricks, V.F., Pedersen, S.A. og Jørgensen, K.F. (eds.) (2003). *Knowledge—Logical Foundations and Applications. Synthese Library Series* (special volume). Kluwer Academic Publishers, forthcoming 2003. Contributions from R. Fagin, J. Halpern, J. Hintikka, W. v. d. Hoek, W. Lenzen, Y. Moses, H. Rott, K. Segerberg, J. Sowa, M. Vardi and R. Wojcicki.

[22] Hintikka, J. (1962). *Knowledge and Belief: An Introduction to the Logic of the Two Notions.* Cornell: Cornell University Press.

[23] Hintikka, J. and M. (1988). *The Logic of Epistemology and the Epistemology of Logic: Selected Essays*. Kluwer Academic Publishers.

[24] Hintikka, J. and Halonen, I. (1998). 'Epistemic Logic,' *Routledge Encyclopedia of Philosophy*, volume 1.

[25] Hintikka, J. (2003). 'A Second Generation Epistemic Logic and its General Significance,' forthcoming in [21].

[26] Hoek, W.v.d. (2003). 'On Epistemic Action Languages,' forthcoming in [21].

[27] Howson, C. and Urbach, P. (1989). *Scientific Reasoning: The Bayesian Approach*. La Salle, Illinois: Open Court.

[28] James, W. (1960). 'The Will to Believe,' *Essays in Pragmatism*. Hafner Publishing Company.

[29] Kelly, K. (1996). *The Logic of Reliable Inquiry.* New York: Oxford University Press.

[30] Kelly, K., Schulte, O. and Hendricks, V. F. (1996). 'Reliable Belief Revision,' *Logic, Methodology, and Philosophy*

of Science X, Chiara, M. L. D. (ed.), Dordrecht: Kluwer Academic Publishers.

[31] Lenzen, W. (1978). *Recent Work in Epistemic Logic*, in *Acta Philosophica Fennica* **30**: 1-219.

[32] Lenzen, W. (2003). 'Knowledge, Belief, and Subjective Probability—Outlines of a Unified System of Epistemic/Doxastic Logic,' forthcoming in [21].

[33] Levi, I. (1997). 'The Logic of Full Belief,' *The Covenant of Reason: Rationality and the Commitments of Thought*, Peters, A. K. (ed.), Cambridge: Cambridge Univesity Press: 40-69.

[34] Lewis, D. (1996). 'Elusive Knowledge,' *Australasian Journal of Philosophy*, 7(1996): 549-567.

[35] Malcolm, N. (1952). 'Knowledge and Belief,' *Mind* LXI, 242. Reprinted in *Contemporary Readings in Epistemology*, Goodman, M. F. and Snyder, R. A. (eds.). Prentice Hall. New Jersey: Englewood Cliffs (1993): 272-279.

[36] Martin, E. & Osherson, D. (1998). *Elements of Scientific Inquiry*. Cambridge: MIT Press.

[37] Meyer, John-Jules Ch. and van der Hoek, Wiebe (1995). *Epistemic Logic for AI and Computer Science*, Cambridge Tracts in Theoretical Computer Science.

[38] Moore, R. C. (1995). *Logic and Represenation,* CSLI Lecture Notes **39**.

[39] Nozick, R. (1981). *Philosophical Explanations*. Cambridge: Harvard University Press.

[40] Nozick, R. (1993). *The Nature of Rationality*. Cambridge: Harvard University Press.

[41] Peirce C. S. (1958). *Charles S. Peirce: Selected Writings*. Ed. Wiener, P. New York: Dover Publications.

[42] Root, H. (2003). 'Interpreting Belief Revision: What is 'economic' in the theory of belief change,' forthcoming in [21].

[43] Sankey, H. (1999). 'A Realist Approach to Method and Truth.' Lecture delivered to the Department of Philosophy, University of Roskilde, Denmark, May 14.

[44] Segerberg, K. (2003). 'Two Attempts at Deconstructing Epistemic Logic,'forthcoming in [21].

[45] Scott, D. (1970). 'Advice on Modal Logic,' in *Philosophical Problems in Logic*, Lambert, K. (ed.). Dordrecht: D. Reidel Publishing Company: 143-173.

[46] Sowa, John F. (2000). *Knowledge Representation: Logical, Philosophical, and Computational Foundations*. Pacific Grove, California: Brooks/Cole.

[47] Sowa, John F. (2003). 'Laws, Facts, and Contexts: Foundations for Multimodal Reasoning,' forthcoming in [21].

[48] Wojcicki, R. (2003). 'Referential Semantics,' forthcoming in [21].

[49] Zanardo, A. (1996). 'Branching-time Logic with Quantification over Branches: The Point of View of Modal Logic,' *Journal of Symbolic Logic*, vol. 61, **1**: 1-39.