

# A First Stab Knowledge Transmissibility and Pluralistic Ignorance

Vincent F. Hendricks  
Department of Philosophy  
University of Copenhagen  
Denmark  
Email: vincent@hum.ku.dk

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Pluralistic ignorance is a nasty informational phenomenon studied widely in social psychology and theoretical economics. It revolves around conditions under which it is "legitimate" for everyone to remain ignorant. In formal epistemology there is enough machinery to model and resolve situations in which pluralistic ignorance may arise. Here is a simple first stab at recovering from pluralistic ignorance by means of knowledge transmissibility.

**Keywords:** *knowledge transmissibility, pluralistic ignorance, epistemic logic, formal learning theory, methodology, information*

In his contribution to *Philosophy of Computing and Information: 5 Questions*, Luciano Floridi explains why he was initially drawn to informational issues:

The second reason was related to what I like to describe as *methodological minimalism*. I was looking for a more "impoverished" approach, which could allow me to work on more elementary concepts, less worn down by centuries of speculation, and more easily manageable. It seemed that, if one could have any hope of answering difficult questions about complicated issues concerning knowledge, meaning, the mind, the nature of reality or morality, it made sense to try to tackle them at the lowest and less committed level at which one could possibly work. Informational and computational ideas provided such a minimalist approach. To give a concrete example, my interest in artificial agents was motivated by the classic idea that

less is more. This is still not very popular among philosophers, who seem to be too much in love with the human subject, his psychology and idiosyncrasies. [Floridi 08]: 94

Floridi's artificial agents may interact—sometimes conducive to inquiry and information aggregation, other times obstructing the very same. Especially social psychologists and economists have for a while both theoretically, using computational tools from observational learning theory, and empirically, through experiments, scrutinized conditions the mechanics of information obstruction while agents interact. Less is more indeed, and techniques from formal epistemology may be wired into these studies providing both models and sometimes resolution. Here is a first stab using an "elementary concept" from epistemic logic, knowledge transmissibility, to tackle pluralistic ignorance and the informational issues related hitherto.

## 1 Pluralistic Ignorance

Pluralistic ignorance may appear when a group of decision-makers have to act or believe at the same time given a public signal [Bikhchandani et al. 98]. When asking a group of new students what they thought was difficult in the reading material handed out for today's lecture chances are good that nobody will signal their comprehension problem. While deciding whether to flag ignorance or not, the individual student firstly and discreetly observes whether the other students encountered the same problem of text comprehension. When all the students are doing this at the same time, the public signal becomes that nobody found the text difficult. Thus in order not to hurt standing, nobody signals ignorance. Everybody refrains from acting on personal information exactly because nobody acts immediately on their personal information. In sum, the danger for pluralistic ignorance arises when the individual decisionmaker in a group lacks the necessary information for solving a problem at hand, and thus observes others hoping for more information. When everybody else does the same, everybody observes the lack of reaction and is consequently lead to erroneous beliefs. This Emperor's New Clothes-mechanism is seen in by-stander effects where people are more likely to intervene in an emergency when alone rather in the presence of others and widely used in sales campaigns and policy making. However, pluralistic ignorance is also fragile as the phenomenon only stands strong as long as nobody cries out. The question—one in sync with Floridi's methodological minimalism: What is the epistemic nature of information leading to bursting the bubble of ignorance?

## 2 Modal Operator Epistemology

The basic formal setup is modal operator epistemology [Hendricks 01], [Hendricks 02], [Hendricks 07] which is the cocktail obtained by mixing formal learning theory

[Kelly 96] and epistemic logic in order to study the formal properties of limiting convergence knowledge:

- An evidence stream  $\varepsilon$  is an  $\omega$ -sequence of natural numbers, *i. e.*,  $\varepsilon \in \omega^\omega$ .
- A possible world has the form  $(\varepsilon, n)$  such that  $\varepsilon \in \omega^\omega$  and the state-coordinate  $n \in \omega$ .
- The set of all possible worlds  $\mathcal{W} = \{(\varepsilon, n) \mid \varepsilon \in \omega^\omega, n \in \omega\}$ .
- $\varepsilon \upharpoonright n$  denotes the finite initial segment of evidence stream  $\varepsilon$  of length  $n$ .
- Define  $\omega^{<\omega}$  to be the set of all finite initial segments of elements in  $\omega$ .
- Let  $(\varepsilon \upharpoonright n)$  denote the set of all infinite evidence streams that extends  $\varepsilon \upharpoonright n$ .
- The set of possible worlds in the fan, *i.e.* background knowledge, is defined as

$$[\varepsilon \upharpoonright n] = (\varepsilon \upharpoonright n) \times \omega.$$

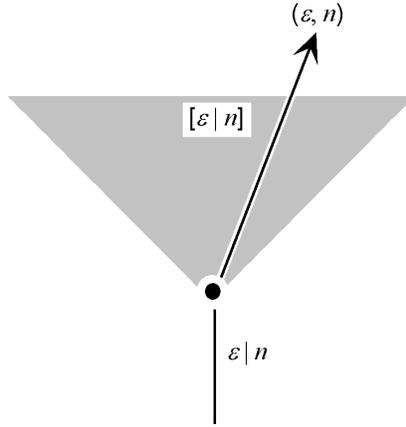


Figure 1: Handle of evidence and fan of worlds

A possible world in the current setup is a significantly different entity from the classical conception. On the traditional view a possible world is not a composite but rather an entity complete in its spatio-temporal history. Here a possible world is a pair consisting of an evidence stream or tape and a state-coordinate allowing one to point to specific entries in the tape, specify all the evidence seen up to a certain point in the inquiry process, quantify over all or some evidence in the evidence tape etc. Thus, what is observed 'right now' is simply the content of a cell in the evidence tape at the specific time 'now' which the state-coordinate determines.

## 2.1 Hypotheses

Hypotheses will be identified with sets of possible worlds. Define the set of all simple empirical hypotheses

$$\mathcal{H} = P(\omega^\omega \times \omega).$$

A hypothesis  $h$  is said to be *true* in world  $(\varepsilon, n)$  iff

$$(\varepsilon, n) \in h \text{ and } \forall l \in \omega : (\varepsilon, n+l) \in h.$$

Truth requires identification and inclusion of the actual world  $(\varepsilon, n)$  in the hypothesis for all possible future states of inquiry.

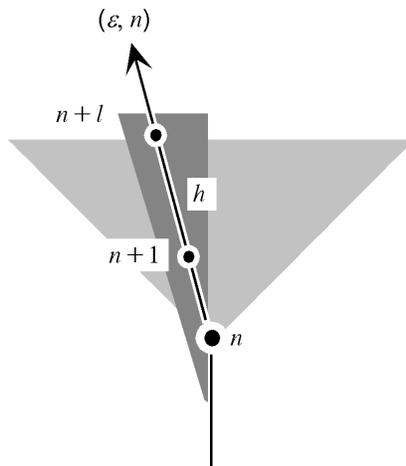


Figure 2: Truth of a hypothesis  $h$  in a possible world  $(\varepsilon, n)$ .

## 2.2 Agents and Inquiry Methods

An inquiry method (or agent) may be either one of discovery or assessment. A discovery method  $\delta$  is a function from finite initial segments of evidence to hypotheses, i.e.

$$\delta : \omega^{<\omega} \longrightarrow \mathcal{H}. \quad (1)$$

The convergence modulus for a *discovery* method (abbreviated *cm*) where  $\mu$  is the least search operator also known as minimalization.

**Definition 1**  $cm(\delta, h, (\varepsilon, n)) = \mu k \forall n' \geq k \forall (\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq h.$

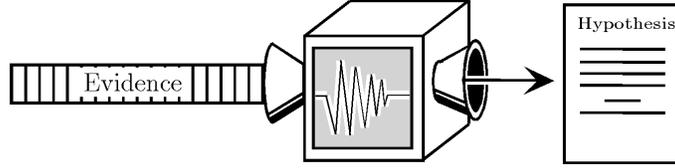


Figure 3: A discovery method  $\delta$

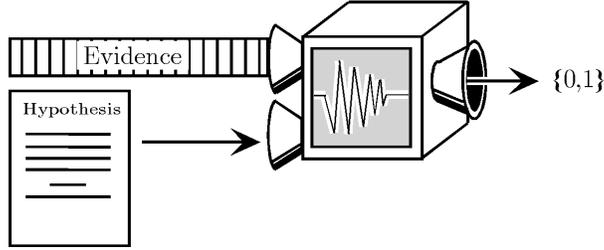


Figure 4: An assessment method  $\alpha$ .

An assessment method  $\alpha$  is a function from finite initial segments of evidence and hypotheses to true/false, i.e.

$$\alpha : \omega^{<\omega} \times \mathcal{H} \longrightarrow \{0, 1\} \quad (2)$$

The convergence modulus for an assessment is defined in the following way:

**Definition 2**  $cm(\alpha, h, (\varepsilon, n)) = \mu k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \varepsilon \mid n) = \alpha(h, \tau \mid n')$ .

### 2.3 Knowledge Based on Discovery

Limiting knowledge for a discovering agent in a possible world requires (1) the hypothesis to be true, (2a) the agent in the limit settles for a conjecture entailing the hypothesis and (2b) the agent does so infallibly in the sense of entailment of the hypothesis by the observed evidence. Formally this condition for limiting convergent discovered knowledge amounts to:

$(\varepsilon, n)$  validates  $K_\delta h$  iff

1.  $(\varepsilon, n) \in h$  and  $\forall l \in \omega : (\varepsilon, n + l) \in h$ ,
2.  $\forall n' \geq n, \forall (\tau, n') \in [\varepsilon \mid n] :$ 
  - (a)  $\delta(\tau \mid n') \subseteq h$ .
  - (b)  $(\tau, n) \in \delta(\tau \mid n')$

The discovery method may additionally be subject to certain agendas (methodological recommendations) like perfect memory, consistency, infallibility etc. [Hendricks 01] but which are of little concern here.

## 2.4 Knowledge Based on Assessment

Limiting knowledge for an assessing agent in a possible world again requires (1) the hypothesis to be true and (2) that the agent in the limit decides the hypothesis in question. This formally comes to:

$(\varepsilon, n)$  validates  $K_\alpha h$  iff

1.  $(\varepsilon, n) \in h$  and  $\forall l \in \omega : (\varepsilon, n + l) \in h$ ,
2.  $\alpha$  decides  $h$  in the limit in  $[\varepsilon \mid n]$  :
  - (a) if  $(\varepsilon, n) \in h$  and  $\forall l \in \omega : (\varepsilon, n + l) \in h$  then  
 $\exists k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \tau \mid n') = 1$ ,
  - (b) if  $(\varepsilon, n) \notin h$  or  $\exists l \in \omega : (\varepsilon, n + l) \notin h$  then  
 $\exists k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha(h, \tau \mid n') = 0$ .

## 3 Multi-Modal Systems

The above set-theoretical characterization of inquiry lends itself to a multi-modal logic. The modal language  $\mathcal{L}$  is defined accordingly:

$$A ::=$$

$$\mid a \mid A \wedge B \mid \neg A \mid K_\delta A \mid K_\alpha A \mid [A!]B \mid I_\delta A \mid I_\alpha A$$

where  $[A!]B$  is the public announcement operator to be read as 'after it has been announced that  $A$ , then  $B$  is the case' [Ditmarsch et al. 2007], [Roy 08] and  $I_\delta A$  is the ignorance operator to be read 'agent  $\delta$  is ignorant of  $A$ ' [Hoek & Lomuscio 04]. In a relational Kripke-model  $[A!]B$  means that after deleting all situations in which  $A$  does not hold in the original models,  $B$  holds. Put differently,  $[A!]B$  takes one from the initial model to a new model; i.e. the accessibility relation is between worlds in different models. Similarly,  $I_\delta A$  says that  $\delta$  is ignorant of  $A$  if there exists two distinct situations or possible worlds, one in which  $A$  is true and yet another one in which  $A$  is false. Operators for alethic as well as tense may also be added to  $\mathcal{L}$ , see [Hendricks 01] and [Hendricks 07] for all details.

### Definition 3 Model

A model  $\mathbb{M} = \langle \mathcal{W}, \varphi, \delta, \alpha \rangle$  consists of:

1. A non-empty set of possible worlds  $\mathcal{W}$ ,
2. A denotation function  $\varphi : \text{Proposition Letters} \longrightarrow P(\mathcal{W})$ , i. e.,  $\varphi(a) \subseteq \mathcal{W}$ .

### 3. Inquiry methods

- (a)  $\delta : \omega^{<\omega} \longrightarrow P(\mathcal{W})$
- (b)  $\alpha : \omega^{<\omega} \times \mathcal{H} \longrightarrow \{0, 1\}$

If  $a$  is a propositional variable, then *semantical correctness* may be defined such that  $(\varepsilon, n) \models_{\mathbb{M}} a$  iff  $\varphi_{\mathbb{M},(\varepsilon,n)}(A) = 1$ . Next, the *semantical meaning* of an arbitrary formula  $A$  in the language may be defined as  $[A]_{\mathbb{M}} = \{(\varepsilon, n) \mid (\varepsilon, n) \models_{\mathbb{M}} A\}$ . The truth conditions for the boolean connectives and remaining operators may be defined as follows:

#### Definition 4 Truth Conditions

Let  $\varphi_{\mathbb{M},(\varepsilon,n)}(A)$  denote the truth value in  $(\varepsilon, n)$  of a modal formula  $A$  given  $\mathbb{M}$ , defined by recursion through the following clauses:

1.  $\varphi_{\mathbb{M},(\varepsilon,n)}(a) = 1$  iff  $(\varepsilon, n) \in \varphi(a)$  and  $\forall l \in \omega : (\varepsilon, n + l) \in \varphi(a)$   
for all propositional variables  $a, b, c, \dots$ .
2.  $\varphi_{\mathbb{M},(\varepsilon,n)}(\neg A) = 1$  iff  $\varphi_{\mathbb{M},(\varepsilon,n)}(A) = 0$ ,
3.  $\varphi_{\mathbb{M},(\varepsilon,n)}(A \wedge B) = 1$  iff both  $\varphi_{\mathbb{M},(\varepsilon,n)}(A) = 1$  and  $\varphi_{\mathbb{M},(\varepsilon,n)}(B) = 1$ ;  
otherwise  $\varphi_{\mathbb{M},(\varepsilon,n)}(A \wedge B) = 0$ .
4.  $\varphi_{\mathbb{M},(\varepsilon,n)}(K_{\delta}A) = 1$  iff
  - (a)  $(\varepsilon, n) \in [A]_{\mathbb{M}}$  and  $\forall l \in \omega : (\varepsilon, n + l) \in [A]_{\mathbb{M}}$ ,
  - (b)  $\forall n' \geq n, \forall (\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq [A]_{\mathbb{M}}$
5.  $\varphi_{\mathbb{M},(\varepsilon,n)}([A!]B) = 1$  iff  
if  $\varphi_{\mathbb{M},(\varepsilon,n)}(A) = 1$ , then  $\varphi_{\mathbb{M},(\varepsilon,n)|A}(B) = 1$ .
6.  $\varphi_{\mathbb{M},(\varepsilon,n)}(I_{\Xi}A) = 1$  iff  $\exists (\tau, m) \exists (\mu, m') \in [\varepsilon \mid n] : \tau \mid n = \mu \mid n$  and  
 $\varphi_{\mathbb{M},(\tau,m)}(A) = 1$  and  $\varphi_{\mathbb{M},(\mu,m')}(\neg A) = 1$  for  $\Xi \in \{\delta, \alpha\}$ .

Model  $\mathbb{M}$ -subscript will be suppressed when it is clear from context. Knowledge based on assessment is omitted from the definition above for reasons of brevity. Note that item 5 introduces a model-restriction  $(\varepsilon, n) \mid A$  similarly to the one introduced in [Roy 08]. Item 6 follows the definition of ignorance defined in [Hoek & Lomuscio 04]. Accessibility is no longer defined as a binary relation on points. Rather two worlds  $(\tau, m)$  and  $(\mu, m')$  are accessible from each other if they have the same handle, i.e.  $\tau \mid n = \mu \mid n$  relative to  $[\varepsilon \mid n]$ .

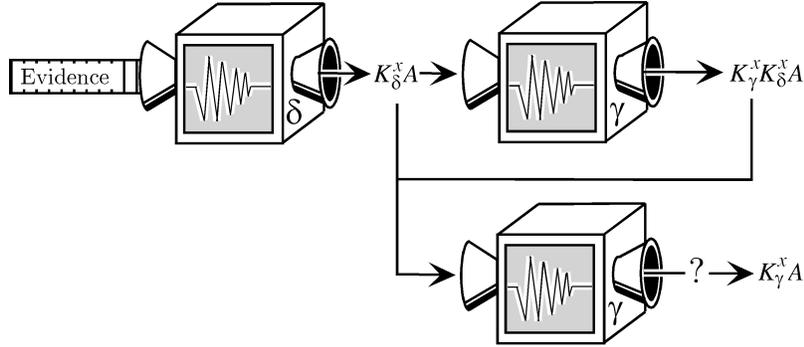


Figure 5: Knowledge transmissibility and inquiry methods.

## 4 Knowledge Transmissibility

Already in *Knowledge and Belief* from Hintikka [Hintikka 62] considered whether

$$K_\beta K_\gamma A \rightarrow K_\beta A \quad (3)$$

is valid (or self-sustainable in Hintikka's terminology) for arbitrary agents  $\beta, \gamma$ . Now 3 is simply an iterated version of Axiom **T** for different agents and as long  $\beta, \gamma$  index the same accessibility relation the claim is straightforward to demonstrate. From an active agent perspective the claim is less obvious.

The reason is agenda-driven or methodological. Inquiry methods  $\beta, \gamma$  may – or may not – be of the same type. If knowledge is subsequently defined either on discovery or assessment, then 3 is not immediately valid unless discovery and assessment methods can "mimic" or induce eachothers' behavior in the following way:

**Theorem 5** *If a discovery method  $\delta$  discovers  $h$  in a possible world  $(\varepsilon, n)$  in the limit, then there exists a limiting assessment method  $\alpha$  which verifies  $h$  in  $(\varepsilon, n)$  in the limit.*

**Proof.** Assume that  $\delta$  discovers  $h$  in  $(\varepsilon, n)$  in the limit and let

$$cm(\delta, h, (\varepsilon, n))$$

be its convergence modulus. Define  $\alpha$  in the following way:

$$\alpha(h, \varepsilon | n) = 1 \text{ iff } \delta(\varepsilon | n) \subseteq h.$$

It is clear that if  $n' \geq cm(\delta, h, [\varepsilon | n])$  then for all  $(\tau, n') \in [\varepsilon | n] : \delta(\tau | n') \subseteq h$ . Consequently  $\alpha(h, \tau | n') = 1$  and therefore

$$cm(\alpha, h, (\varepsilon, n)) \leq cm(\delta, h, (\varepsilon, n)).$$

■

Similarly, but conversely:

**Theorem 6** *If an assessment method  $\alpha$  verifies  $h$  in  $(\varepsilon, n)$  in the limit, then there exists a limiting discovery method  $\delta$  which discovers  $h$  in  $(\varepsilon, n)$  in the limit.*

**Proof.** Similar construction as in proof of Theorem 5. ■

Using inducement it is easily shown that:

**Theorem 7**  $K_\delta A \leftrightarrow K_\alpha A$ .

**Proof.**

( $\rightarrow$ ) Show that  $(\varepsilon, n) \models K_\delta A \rightarrow K_\alpha A$ . Then

- (1)  $(\varepsilon, n) \in [A]$  and  $\forall l \in \omega : (\varepsilon, n + l) \in [A]$
- (2)  $\forall n' \geq n, \forall (\tau, n') \in [\varepsilon \mid n]$ :
- (2.a)  $\delta(\tau \mid n') \subseteq [A]$ ,
- (2.b)  $(\tau, n) \in \delta(\tau \mid n')$ .

Let discovery method  $\delta$  induce its assessment correlate  $\alpha$  in accordance with theorem 5. Since  $(\varepsilon, n) \in [A]$  and  $\forall l \in \omega : (\varepsilon, n + l) \in [A]$  by (1) construct the assessment correlate in such a way that

$$\forall n' \geq n, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha([A], \tau \mid n') = 1 \leftrightarrow \delta(\tau \mid n') \subseteq [A].$$

But by (2)  $\forall n' \geq n, \forall (\tau, n') \in [\varepsilon \mid n] : \delta(\tau \mid n') \subseteq [A]$  so

$$\exists k \geq n, \forall n' \geq k, \forall (\tau, n') \in [\varepsilon \mid n] : \alpha([A], \tau \mid n') = 1$$

which amounts to the definition of knowledge based on assessment.

( $\leftarrow$ ) Show that  $(\varepsilon, n) \models K_\delta A \leftarrow K_\alpha A$ . This proof is similar to ( $\rightarrow$ ) observing theorem 6.

■

In [Hendricks 01] it is proved that knowledge based on discovery and assessment under suitable methodological recommendations validate **S4**. Hintikka's original knowledge transmissibility thesis 3 may be easily proved now since theorems 5, 6 and 7 provide the assurance that a discovery method can do whatever an assessment method can do and vice versa:

$$\begin{array}{lll} \vdash K_\delta K_\alpha A \rightarrow K_\delta A & & \\ (i) & K_\alpha A \rightarrow A & \text{Axiom } \mathbf{T} \\ (ii) & K_\delta(K_\alpha A \rightarrow A) & (i), (N) \\ (iii) & K_\delta(K_\alpha A \rightarrow A) \rightarrow & (ii), \text{Axiom } \mathbf{K} \\ & (K_\delta K_\alpha A \rightarrow K_\delta A) & \\ (iv) & K_\delta K_\alpha A \rightarrow K_\delta A & (ii), (iii), (MP) \end{array}$$

In the literature one often encounters defences for epistemic norms derived from 3 like:

**Conjecture 8** *If  $\delta$  knows that  $A$ , then  $\delta$  has a certain degree of evidence for  $A$ , or  $\delta$  has had  $A$  transmitted from  $\gamma$  who has evidence for  $A$ .*

This may indeed be so, but "evidence for  $A$ " is essentially an issue related to how the evidence or information has been collected—thus an agenda concern, or a methodological concern, of how inquiry methods for knowledge acquisition interact.

## 5 Knowledge Transmissibility and Public Announcement

In order to study knowledge transmissibility and pluralistic ignorance, the relation between transmissibility and public announcement have to be uncovered first. Here is a set of theorems relating knowledge transmissibility to public announcement. Proofs are omitted but may be found in [Hendricks 11].

Let there be given a finite set of discovery agents  $\Delta = \{\delta_1, \delta_2, \delta_3, \dots, \delta_n\}$ , a finite set of assessment agents  $\Lambda = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$  and assume that all agents in  $\Delta, \Lambda$  has **S4**-knowledge. Now it may be shown that 3 holds for agents of different types:

**Theorem 9**  $\forall \delta_i \in \Delta : K_{\delta_i} K_{\alpha} A \rightarrow \bigwedge_{i=1}^n K_{\delta_i} A$  if theorem 5 holds for  $\Delta = \{\delta_1, \delta_2, \delta_3, \dots, \delta_n\}$ .

**Theorem 10**  $\forall \alpha_i \in \Lambda : K_{\alpha_i} K_{\delta} A \rightarrow \bigwedge_{i=1}^n K_{\alpha_i} A$  if theorem 6 holds for  $\Lambda = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ .

The next two theorems show that the axiom relating public announcement to knowledge given the standard axiomatization of public announcement logic with common knowledge [Ditmarsch et al. 2007] holds for knowledge based on discovery and knowledge based on assessment. This is a variation of the original knowledge prediction axiom which states that "some  $a$  knows  $B$  after an announcement  $A$  iff (if  $A$  is true,  $a$  knows that after the announcement of  $A$ ,  $B$  will be the case)":

**Theorem 11**  $\forall \delta_i \in \Delta : [K_{\alpha} A!] K_{\delta_i} B \leftrightarrow (K_{\alpha} A \rightarrow (K_{\delta_i} K_{\alpha} A \rightarrow [K_{\alpha} A!] B))$  if theorem 2 holds for  $\Delta = \{\delta_1, \delta_2, \delta_3, \dots, \delta_n\}$ .

**Theorem 12**  $\forall \alpha_i \in \Lambda : [K_{\delta} A!] K_{\alpha_i} B \leftrightarrow (K_{\delta} A \rightarrow (K_{\alpha_i} [K_{\delta} A \rightarrow [K_{\delta} A!] B]))$  if theorem 3 holds for  $\Lambda = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ .

## 6 Knowledge Transmissibility and Pluralistic Ignorance

Recall that pluralistic ignorance may appear when the individuals in a group lack the required information for solving a current problem, and thus observes others

hoping for more information. When everybody else does the same, everybody observes the lack of reaction and is consequently lead to erroneous beliefs. A simple situation of pluralistic ignorance arises in H.C. Andersen's fable, *The Emperor's New Clothes*, in which a small boy dispells the spell of pluralistic ignorance by publicly announcing the lack of garments on the emperor. To model this particular situation, assume:

1. A finite set ignorant agents either based on discovery of assessment or both:

(a)  $\Delta = \{\delta_1, \delta_2, \delta_3, \dots, \delta_n\}$

(b)  $\Lambda = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$

2. A public announcement such that after it has been announced that  $A$ , then  $B$  is the case:

(a)  $[A!]B$

Assume further more the existence of

1. At least one knowing agent based on either discovery or assessment:

(a)  $K_\alpha A$

(b)  $K_\delta A$

2. Inducement theorems 5 and 6.

Pluralistic ignorance based on discovery or assessment may established in the following way:

3. Discovery:  $\forall \delta_i \in \Delta : [C!]I_{\delta_i} A$ .

4. Assessment:  $\forall \alpha_i \in \Lambda : [C!]I_{\alpha_i} A$

where  $C$  is the public signal (announcement) leading all agents in  $\Delta$  or  $\Lambda$  to ignorance of  $A$ . The airport announces  $C$ : 'Some flights are delayed' and every  $\delta_i \in \Delta$  become ignorant as to  $A$ : 'My flight is delayed' or  $\neg A$ : 'My flight is not delayed'. Or the teacher publicly announces  $C$ : 'This exercise is tricky' and every student  $\forall \alpha_i \in \Lambda$  becomes ignorant as to  $A$ : 'I got it right' or  $\neg A$ : 'I got it wrong'.

Suppose there is a knowing agent (the little boy in *The Emperor's New Clothes*) such that

5.  $K_\alpha A$  for bullet 3.

6.  $K_\delta A$  for bullet 4.

Now it is possible to prove the following two theorems.

**Theorem 13**  $\forall \delta_i \in \Delta : I_{\delta_i} A \wedge ([K_{\alpha} A]!A \wedge K_{\delta_i} K_{\alpha} A) \rightarrow \bigwedge_{i=1}^n K_{\delta_i} A$

if theorem 5 holds for  $\Delta = \{\delta_1, \delta_2, \delta_3, \dots, \delta_n\}$ .

The theorem conveys that if

- it holds for all agents  $\delta_i \in \Delta$  that they are ignorant of  $A$  based on  $\forall \delta_i \in \Delta : [C!]I_{\delta_i} A$ , and
- that after it has been publicly announced that  $\alpha$  knows  $A$ , then  $A$  is the case, and
- all agents  $\delta_i \in \Delta$  knows that  $\alpha$  knows  $A$ , then
- $\alpha$ 's knowledge of  $A$  will be transferred to every  $\delta_i \in \Delta$  provided that
- every  $\delta_i \in \Delta$  can mimic  $\alpha$ 's epistemic behavior based on theorem 5.

**Proof.**

See [Hendricks 11].

■

**Theorem 14**  $\forall \alpha_i \in \Lambda : I_{\alpha_i} A \wedge ([K_{\delta} A]!A \wedge K_{\alpha_i} K_{\delta} A) \rightarrow \bigwedge_{i=1}^n K_{\alpha_i} A$

if theorem 6 holds for  $\Lambda = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ .

This theorem says that if

- it holds for all agents  $\alpha_i \in \Lambda$  that they are ignorant of  $A$  based on  $\forall \alpha_i \in \Lambda : [C!]I_{\alpha_i} A$ , and
- that after it has been publicly announced that  $\delta$  knows  $A$ , then  $A$  is the case, and
- all agents  $\alpha_i \in \Lambda$  knows that  $\delta$  knows  $A$ , then
- $\delta$ 's knowledge of  $A$  will be transferred to every  $\alpha_i \in \Lambda$  provided that
- every  $\alpha_i \in \Lambda$  can mimic  $\delta$ 's epistemic behavior based on theorem 6.

**Proof.**

See [Hendricks 11].

■

Besides the formalization of pluralistic ignorance, the latter theorems demonstrate how some simple versions of pluralistic ignorance may be dispelled using an elementary, and perhaps also less shopworn notion, from epistemic logic which fits well with Floridi's wish for "methodological minimalism" and "artificial agents" with emphasis on computing and information-driven inquiry.

There is so much more of this to be done here, and it is certainly worth our epistemological time – mainstream and formal alike – to consider the interesting social phenomena uncovered by social psychology and behavior economics. This was a first stab.

## 7 Acknowledgements

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