

Proof theory, History and philosophical significance, edited by Vincent F. Hendricks, Stig Andur Pedersen, and Klaus Frovin Jørgensen, Synthese library, vol. 292, Kluwer Academic Publishers, Dordrecht, Boston, and London, 2000, xii + 244 pp.—therein:

VINCENT F. HENDRICKS, STIG ANDUR PEDERSEN, and KLAUS FROVIN JØRGENSEN. *Introduction*. Pp. 1–7.

SOLOMON FEFERMAN. *Highlights in proof theory*. Pp. 11–31.

LEO CORRY. *The empiricist roots of Hilbert's axiomatic approach*. Pp. 35–54.

DAVID ROWE. *The calm before the storm: Hilbert's early views on foundations*. Pp. 55–93.

WILFRIED SIEG. *Toward finitist proof theory*. Pp. 95–114.

DIRK VAN DALEN. *The development of Brouwer's intuitionism*. Pp. 117–152.

MORITZ EPPLE. *Did Brouwer's intuitionistic analysis satisfy its own epistemological standards?* Pp. 153–178.

SOLOMON FEFERMAN. *The significance of Weyl's Das Kontinuum*. Pp. 179–194.

ERHARD SCHOLZ. *Hermann Weyl on the concept of continuum*. Pp. 195–217.

SOLOMON FEFERMAN. *Relationships between constructive, predicative and classical systems of analysis*. Pp. 221–236.

This collection of papers is based on a conference with the same title held at the University of Roskilde, Denmark, in 1997. Three systematic lectures by Feferman form the backbone of the volume. Their texts maintain the spontaneity of a lecture without sacrificing accuracy of detail and are a pleasure to read. The first one traces the path from Hilbert's old axiomatic proof theory to Gentzen's structural proof theory as based on sequent calculi, and ends up with a brief review of infinitary systems of proof.

Feferman's first essay is the only one dealing with developments in proof theory from the beginning of the 1930's on, except for a brief mention of Gödel and Gentzen by Sieg. Despite its title and the wish of the editors to "trace the development of proof theory and illuminate its growth into a mature theory" (*Introduction*, p. 1), the contents of the volume might be described better as "Hilbert's old proof theory and the intuitionism of Brouwer and Weyl." All the essays except the first and last ones, both by Feferman, fall entirely within this category. As an indication of the limited coverage of proof theory, it can be noted that such a great success of proof-theoretical research of the last decades as the computational semantics of intuitionistic or constructive logic, also known as the "Curry–Howard isomorphism," does not get as much as a mention in the volume.

The foundational work of Hilbert, Weyl, and Brouwer has received a large amount of attention ever since the "Grundlagenstreit" of the 1920's. It should be safe to say that there is a general understanding of the positions of the main protagonists of the early foundational debates. Beyond a general understanding, detailed historical-foundational work is an entirely different matter and requires, besides specialized knowledge in logic and mathematics, experience in the methods of historical research. Such detailed work for the period in question is still largely to be done; for example, work on the Hilbert school of the 1920's is quite recent, and is exemplified in this volume by the essay of Wilfried Sieg.

The editors open their *Introduction* by describing proof theory as "a technical sub-field of mathematical logic." This should hardly be the attitude of most proof theorists who view their work as part of a distinctly foundational enterprise: the study of the structure of mathematical proofs. Classic examples of foundational results, in contrast to "technical" (whatever that may be), include the full mastery of the structure of proofs in first-order logic through Gentzen's sequent calculus, and the consistency of arithmetic and the determination of its proof-theoretic strength. There is any amount of subsequent work along similar lines, "technical" only insofar as that is considered a synonym for difficult.

This volume has a thematic unity that places it above most other books originating in conference papers. Its usefulness for those interested in the earlier phases of the foundational debate is further increased by a detailed index, for which the editors deserve a sincere thanks.

JAN VON PLATO

Department of Philosophy, University of Helsinki, SF-00014 Helsinki, Finland.
vonplato@helsinki.fi.